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FAST KEMENY CONSENSUS BY SEARCHING OVER STANDARD MATRICES DISTANCED TO THE AVERAGED EXPERT RANKING BY MINIMAL DIFFERENCE

Background. The problem of ranking a finite set of objects is considered.

Objective. The goal is to develop an algorithm that would let speed up the search of the Kemeny consensus along with substantiation of a metric to compare rankings.

Methods. An approach for aggregating experts' rankings is suggested and substantiated. Also a metric to compare rankings is suggested and substantiated.

Results. The developed algorithm finds a set of Kemeny rankings much faster than the classical straightforward search. Also this set often contains a single Kemeny consensus, what fails by the straightforward search. Besides, a single Kemeny consensus is determined at one stroke if the averaged expert ranking turns out acyclic. Thus the problem of selecting a single Kemeny consensus is solved.

Conclusions. For 10 objects and more, where most known approaches become intractable, the algorithm still is tractable due to searching over only those standard matrices whose distance to the first ranking differs minimally from the distance between this ranking and the averaged expert ranking.

Keywords: ranking; Kemeny consensus; averaged expert ranking.

Introduction

Ranking objects is an important task arising along with a lot of technical and social-economic problems. These problems are called for multicriteria optimization, dispatching priorities, distribution, resources allocation, voting schemes, etc. Herein one deals with consensus problems where various and contradictory demands are tried to be satisfied. Thus a consensus is searched over a finite set of permutations each of which shows a ranking, and this consensus should be as close as possible to a set of rankings given by experts or voters. The problem of determining the consensus ranking known as Kemeny consensus (or Kemeny ranking) is NP-hard [1, 2]. For instance, if there are 4 objects then altogether we have 24 possible consensus versions, but 10 objects give us 3628800 versions. Note that here those ones are considered as acyclic rankings. If consider any rankings then 4 objects give 64 versions, and 10 objects generate 35184372088832 versions (more than 35184 billions). For now, according to [3, 4], computational complexity of the Kemeny consensus is reduced down to polynomial-time approximation algorithms including a deterministic algorithm [5] and a randomized algorithm (see the reference in [1]). A polynomial-time approximation scheme was developed in [6], although its running time turned out completely impractical. Greedy and branch-and-bound heuristic approaches were used in [7, 8] for the efficient exact computation of a Kemeny consensus. A broad study of the parameterized complexity for computing optimal Kemeny rankings was provided in [1]. Many other approaches treat similarities among objects so that it would help to efficiently compute Kemeny rankings [9, 10]. Nevertheless, without any additional conventions, Kemeny rankings are determined by straightforward search applying Kemeny–Snell distance or Kendall tau ranking distance [1, 11, 12]. For speeding up the computational process, heuristics and approximations are applied as well. And another question relates to heuristic initialization. This is about the choice of the distance function, concerning also its inputs. The matter is that the distance to the set of experts' rankings can be treated differently.

Problem statement

Issuing from the Kemeny consensus is determined straightforwardly too long and the ambiguity about measuring differences exists, the goal is to develop an algorithm that would let speed up the search along with substantiation of a metric to compare rankings. Besides, the algorithm should ensure its tractability for at least 10 objects. To reach the declared goal, the following three items are going to be fulfilled:

1. Represent a visualization of the general routine of determining the Kemeny consensus.
2. Suggest and substantiate an approach for aggregating experts' rankings.
3. Suggest and substantiate a metric to compare rankings.

4. Develop an algorithm that would let speed up searching over the set of the given (standard) rankings.
5. Compare running times of the developed algorithm and the classical straightforward search.
6. Discuss the obtain results and make a conclusion on them.

The general routine of determining the Kemeny consensus

When a finite set of N objects by $N \in \mathbb{N}\backslash\{1\}$ is ranked to a strict order, these objects are compared pairwise by J experts at $J \in \mathbb{N}\backslash\{1\}$ who give a set of J matrices $\{\mathbf{B}_j\}_{j=1}^{J}$ where $\mathbf{B}_j = [b_{ik}^{\langle j \rangle}]_{N\times N}$ by $\mathbf{B}_j = -\mathbf{B}_j^{\mathrm{T}}$ and, specifically, $\mathbf{B}_j \in \mathscr{H}_N$ by the $N\times N$ matrix space

$$\mathscr{H}_N = \{\mathbf{H} = [h_{ik}]_{N\times N} : h_{ik} = -h_{ki},$$
$$h_{ik} \in \{-1, 1\} \text{ by } i \neq k\}. \tag{1}$$

If $b_{ik}^{\langle j \rangle} = 1$ then by the j-th expert's judgment, conventionally, the rank of the i-th object is higher than the rank of the k-th object. And vice versa, $b_{ik}^{\langle j \rangle} = -1$ if the rank of the i-th object is lower than the rank of the k-th object. Every matrix in the space (1) reflects the same logic.

Each expert may have a factor of its own competence assigned before the expert procedure starts. The j-th expert's competence is a positive value ξ_j which is

$$\xi_j \in (0;1) \text{ for } j = \overline{1, J} \text{ by } \sum_{j=1}^{J} \xi_j = 1.$$

Subsequently, the experts' rankings $\{\mathbf{B}_j\}_{j=1}^{J}$ are weighted with factors $\{\xi_j\}_{j=1}^{J}$. If there is no information about the experts' competences or these data are unreliable (unavailable), then the set $\{\xi_j\}_{j=1}^{J}$ is ignored and the rankings $\{\mathbf{B}_j\}_{j=1}^{J}$ are not weighted. Sometimes this case is said that the experts have identical competences.

A Kemeny consensus is a matrix $\tilde{\mathbf{G}}^* = [\tilde{g}_{ik}^*]_{N\times N}$ by $\tilde{\mathbf{G}}^* \in \mathscr{H}_N$. If not only acyclic rankings are admissible, then the whole set of all possible rankings coincides with the space (1) which has $2^{\frac{N(N-1)}{2}} = \sqrt{2^{N(N-1)}}$ elements. In more specific fields of study, a Kemeny consensus must be determined within a set $\{\mathbf{G}_r\}_{r\in S}$ by $\mathbf{G}_r = [g_{ik}^{\langle r \rangle}]_{N\times N}$ and $\mathbf{G}_r \in \mathscr{H}_N$ where the set of indices $S \subset \left\{\overline{1, \sqrt{2^{N(N-1)}}}\right\}$ and, certainly, $|S| > 1$. Unlike rankings $\{\mathbf{B}_j\}_{j=1}^{J}$ with availability of the factors $\{\xi_j\}_{j=1}^{J}$, matrices within the set $\{\mathbf{G}_r\}_{r\in S}$ do not have any weights (Fig. 1).

By the classical straightforward search, the Kemeny consensus $\tilde{\mathbf{G}}^*$ is an element of the set [13, 14]

$$\arg\min_{\{\mathbf{G}_r\}_{r\in S}} \sum_{j=1}^{J} \xi_j \rho(\mathbf{B}_j, \mathbf{G}_r) \tag{2}$$

if experts have non-identical competences, and is an element of the set

$$\arg\min_{\{\mathbf{G}_r\}_{r\in S}} \sum_{j=1}^{J} \rho(\mathbf{B}_j, \mathbf{G}_r) \tag{3}$$

if their competences are identical, where Kemeny–Snell distance or Kendall tau ranking distance both

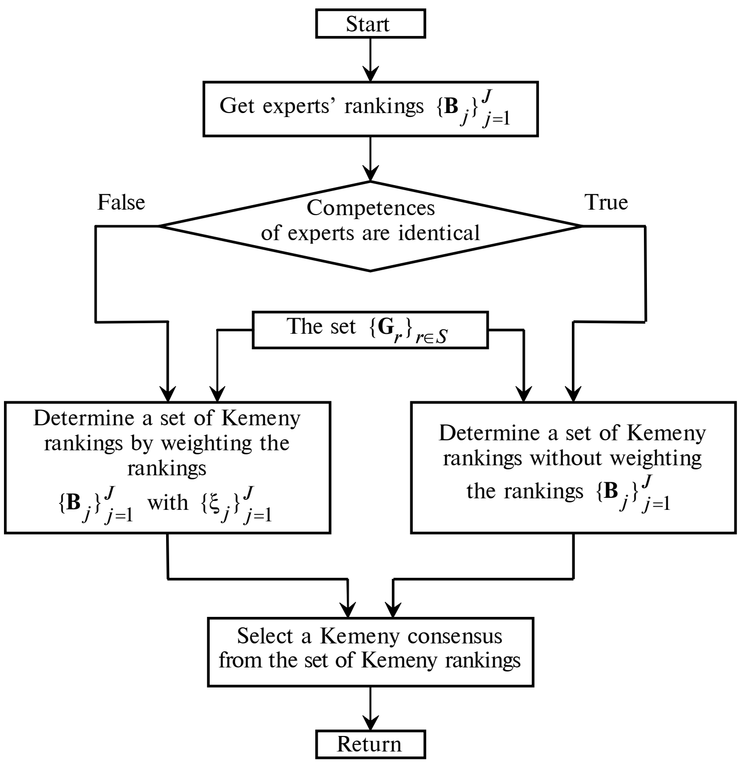

Fig. 1. The Kemeny consensus is determined by either weighting the experts' rankings or not

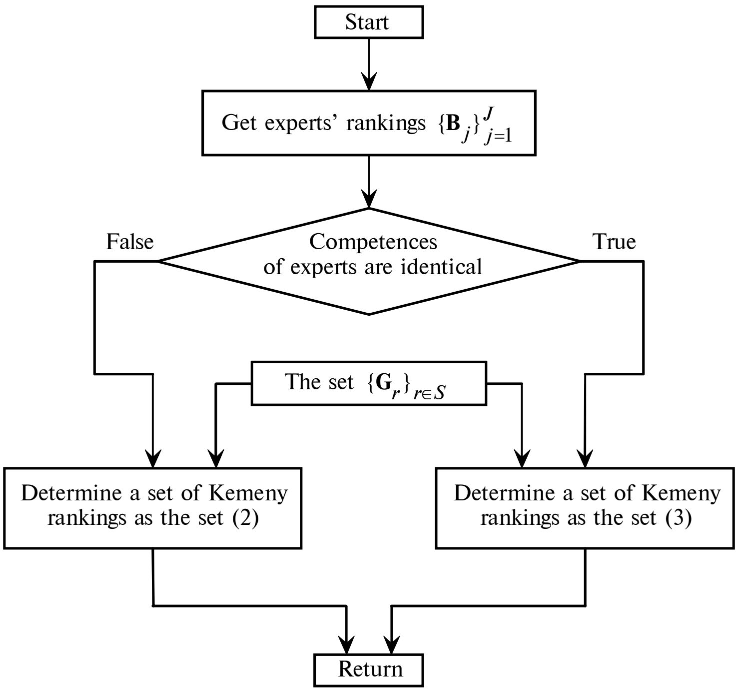

Fig. 2. Classical straightforward search where Kemeny–Snell distance or Kendall tau ranking distance is applied

denoted by $\rho(\mathbf{B}_j, \mathbf{G}_r)$ is applied to measure the difference between matrices $\mathbf{B}_j$ and $\mathbf{G}_r$ (Fig. 2). Practically, when the number of experts is not great, it is observed that the set (2) tends to have fewer Kemeny rankings than the set (3) has. Scanty groups of experts without information about their competences generate the problem of selecting a single Kemeny consensus from two or more Kemeny rankings.

Commonly, the general routine visualized in Fig. 1 and rendered into an explicit search in Fig. 2 does not always calculate the single Kemeny consensus. Another peculiarity is that the aggregation of the experts' rankings appears implicit. This is because the aggregation is expected to be a matrix $\tilde{\mathbf{B}} = [\tilde{b}_{ik}]_{N\times N}$ by $\tilde{\mathbf{B}} \in \mathscr{H}_N$ which is processed subsequently.

An approach for aggregating experts' rankings

A plain and simple approach for aggregating experts' rankings consists in ordinary averaging. The averaged expert matrix $\bar{\mathbf{B}} = [\bar{b}_{ik}]_{N\times N}$ is either

$$\bar{\mathbf{B}} = \sum_{j=1}^{J} \xi_j \mathbf{B}_j \qquad (4)$$

or

$$\bar{\mathbf{B}} = \frac{1}{J}\sum_{j=1}^{J} \mathbf{B}_j \,. \qquad (5)$$

It is apparent that, generally, $\bar{\mathbf{B}} \notin \mathscr{H}_N$. But with obvious mapping

$$\tilde{b}_{ik} = \operatorname{sign} \bar{b}_{ik} \quad \forall\, i = \overline{1, N} \text{ and } \forall\, k = \overline{1, N} \qquad (6)$$

we get the matrix $\tilde{\mathbf{B}} \in \mathscr{H}_N$. The averaged expert ranking $\tilde{\mathbf{B}}$ allows to repudiate the weighted distances under minimum in (2) and (3). Application of them implies measuring differences between the set $\{\mathbf{B}_j\}_{j=1}^J$ and each element in the set $\{\mathbf{G}_r\}_{r\in S}$, though comparing $\tilde{\mathbf{B}}$ and $\mathbf{G}_r$ looks more natural. Besides, the averaged expert ranking $\tilde{\mathbf{B}}$ can turn out acyclic giving the Kemeny consensus $\tilde{\mathbf{G}}^* = \tilde{\mathbf{B}}$ at one stroke. Therefore, the approach for aggregating experts' rankings by (4), (5), and (6) is reasonable. And instead of the weighted distance, the other distance is going to be applied.

A metric for comparing rankings

Straightforwardly comparing $\tilde{\mathbf{B}}$ and $\mathbf{G}_r$ for all $r \in S$ by $S = \left\{\overline{1, \sqrt{2^{N(N-1)}}}\right\}$ takes too long time and is intractable itself for $N > 9$. If acyclic rankings are only admissible as standard ones, then let the subspace of all acyclic rankings be $\bar{\mathscr{H}}_N \subset \mathscr{H}_N$ and the search must be just within the subspace $\bar{\mathscr{H}}_N \subset \mathscr{H}_N$. For further convenience, enumerate elements of the space (1), without loss of generality, so that

$$\mathbf{G}_1 = [g_{ik}^{\langle 1\rangle}]_{N\times N} \text{ by } g_{ik}^{\langle 1\rangle} = 1$$

$$\forall i = \overline{1, N-1} \text{ and } \forall\, k = \overline{i+1, N}\,. \qquad (7)$$

Obviously, $\mathbf{G}_1 \in \bar{\mathscr{H}}_N$ what corresponds to the case when the objects are ranked according to their numbers. It is naively manifest to measure the difference between $\tilde{\mathbf{B}}$ and $\mathbf{G}_1$. This difference should be close to the difference between $\mathbf{G}_1$ and $\tilde{\mathbf{G}}^*$. Henceforward, if $\mathbf{X} = [x_{ik}]_{N\times N} \in \mathscr{H}_N$ and $\mathbf{Y} = [y_{ik}]_{N\times N} \in \mathscr{H}_N$ then let

$$\rho_{-1}(\mathbf{X}, \mathbf{Y}) = \frac{1}{4}\sum_{i=1}^{N}\sum_{k=1}^{N} | x_{ik} - y_{ik} | = \frac{1}{2}\sum_{i=1}^{N-1}\sum_{k=i+1}^{N} | x_{ik} - y_{ik} | . \quad (8)$$

The metric (8) in the space (1) is the distance similar to the Kendall tau ranking distance, where just the amount of mismatches between the matrices' entries is counted up. This amount fits natively for comparing rankings.

An algorithm with the distance to the first ranking (7)

If $\tilde{\mathbf{B}} \notin \bar{\mathscr{H}}_N$ then the distance

$$d_1 = \rho_{-1}(\tilde{\mathbf{B}}, \mathbf{G}_1) \quad (9)$$

can be interpreted as an indicator at a point in the subspace $\bar{\mathscr{H}}_N$, around which the desired consensus is expected to be close enough. Primarily, it would have been sufficient to gather all the matrices

$$\{\mathbf{G}_t\}_{t\in T\subset S} \subset \{\mathbf{G}_r\}_{r\in S} = \bar{\mathscr{H}}_N \quad (10)$$

such that

$$\rho_{-1}(\mathbf{G}_t, \mathbf{G}_1) = d_1 \quad \forall t \in T \subset S . \quad (11)$$

However, the equation (11) with respect to the matrix $\mathbf{G}_t$ produces a narrower subset of rankings than that which may be needed to include the consensus $\tilde{\mathbf{G}}^*$. This is why, instead of the equation (11), we gather all the matrices (10) such that

$$\rho_{-1}(\mathbf{G}_t, \mathbf{G}_1) \in \{d_1 - 1, d_1, d_1 + 1\} \quad \forall t \in T \subset S . \quad (12)$$

As soon as the subset (10) by (12) is formed, the subset

$$T^* = \arg\min_{t\in T\subset S} \rho_{-1}(\mathbf{G}_t, \tilde{\mathbf{B}}) \quad (13)$$

of indices is found. Factually, the Kemeny consensus

$$\tilde{\mathbf{G}}^* = \mathbf{G}_{t^*} \text{ by } t^* \in T^* \subset T \subset S \quad (14)$$

if the subset $T^* \subset T$ has just one index. If not, i.e. $| T^* | > 1$, then the Kemeny consensus must be sifted out from the subset $\{\mathbf{G}_{t^*}\}_{t^*\in T^*}$ by applying a supplementary criterion. Such a criterion comes to be the distance between the set $\{\mathbf{B}_j\}_{j=1}^{J}$ and each element in the set $\{\mathbf{G}_{t^*}\}_{t^*\in T^*}$. Thus,

$$\tilde{T}^* = \arg\min_{t^*\in T^*} \sum_{j=1}^{J} \xi_j \rho_{-1}(\mathbf{B}_j, \mathbf{G}_{t^*}) \quad (15)$$

if experts have non-identical competences, and

$$\tilde{T}^* = \arg\min_{t^*\in T^*} \sum_{j=1}^{J} \rho_{-1}(\mathbf{B}_j, \mathbf{G}_{t^*}) \quad (16)$$

if their competences are identical, where the subset $\tilde{T}^*$ contains indices of Kemeny rankings optimal by the minimized difference between each of them and the set $\{\mathbf{B}_j\}_{j=1}^{J}$. If the subset $\tilde{T}^* \subset T^* \subset T$ has just one index, herein the Kemeny consensus

$$\tilde{\mathbf{G}}^* = \mathbf{G}_{\tilde{t}^*} \text{ by } \tilde{t}^* \in \tilde{T}^* \subset T^* \subset T \subset S \quad (17)$$

comes single (Fig. 3).

It is not excluded that the subsets T^* and $\tilde{T}^*$ happen to be the same. Especially when $| T^* | = 1$ and the Kemeny consensus is (14), needing not finding (15) or (16). It is nonetheless apparent that possibility to obtain a single Kemeny consensus according to (17) is much stronger than obtaining a single Kemeny consensus just according to (14).

Gains in running times and distances to the aggregation of the experts' rankings

Denote by $\tau(N, J)$ the running time by the straightforward search, and denote by $\tau_1(N, J)$ the running time by the algorithm schemed in Fig. 3. The running time gain is

$$\gamma(N, J) = \frac{\tau(N, J)}{\tau_1(N, J)} . \quad (18)$$

For a few objects and experts, the gain (18) is reverse. But $\gamma(N, J) > 1$ by either $N > 8$ or by $N > 5$ at $J > 50$. Amazingly enough, 10 objects are ranked in 10 minutes, where the gain $\gamma(10, J) > 30$ for any J. The gain $\gamma(10, J)$ increases with increasing the number of experts.

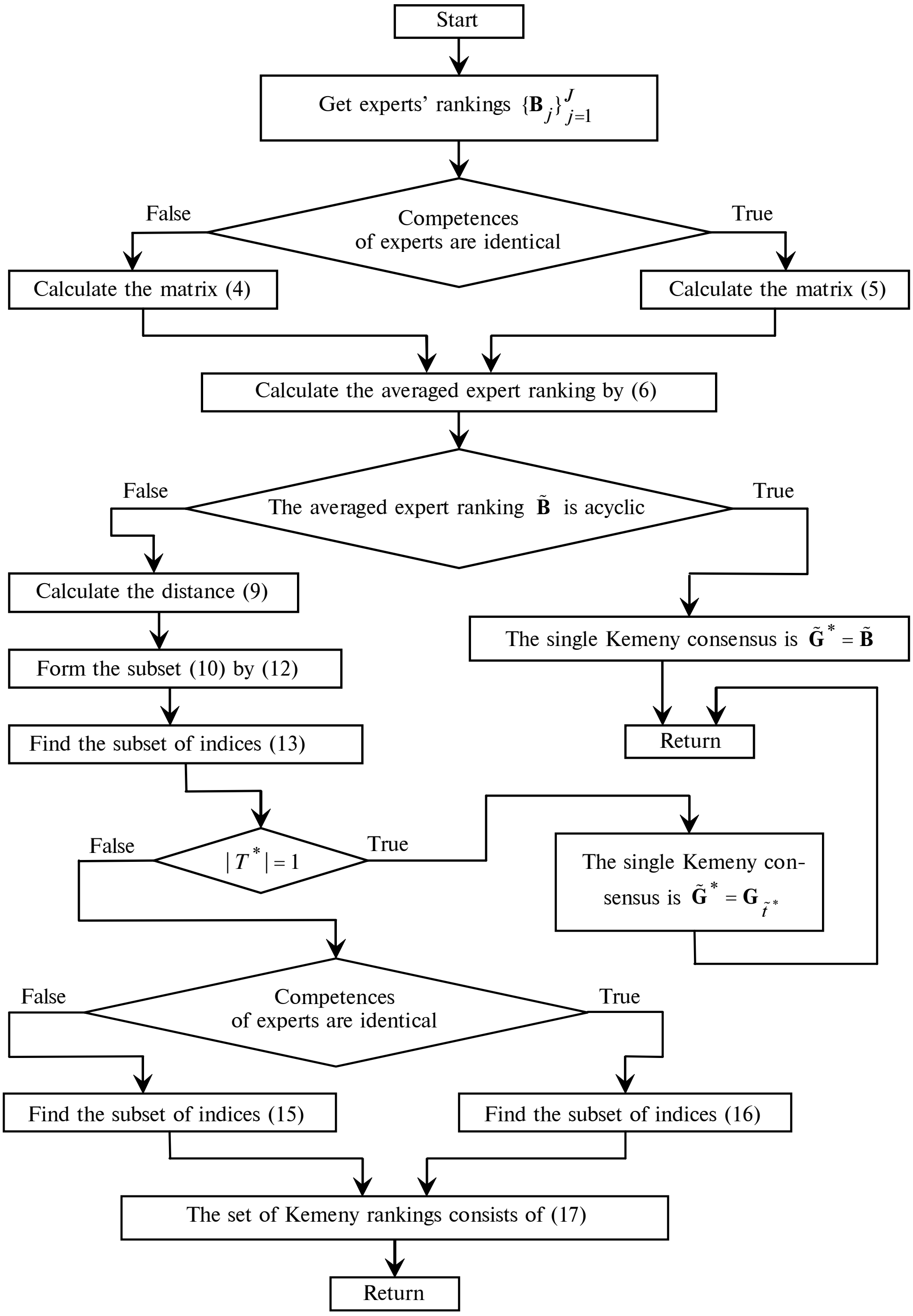

Fig. 3. An algorithm trying to sift out a single Kemeny consensus from the subset of standard rankings whose distance to the first ranking (7) differs from the averaged expert ranking no more than by 1

A very important feature is the minimized score

$$\tilde{d}_J^* = \min_{\tilde{t}^* \in T^*} \sum_{j=1}^{J} \xi_j \rho_{-1}(\mathbf{B}_j, \mathbf{G}_{\tilde{t}^*}) \qquad (19)$$

where the set $\{\xi_j = J^{-1}\}_{j=1}^{J}$ is taken if experts have identical competences. The score (19) stands against the Kemeny score

$$d_J^{\langle S \rangle} = \min_{r \in S} \sum_{j=1}^{J} \xi_j \rho_{-1}(\mathbf{B}_j, \mathbf{G}_r) . \qquad (20)$$

Despite the relationship between (19) and (20) is the clear inequality $\tilde{d}_J^* \geqslant d_J^{\langle S \rangle}$, the developed algorithm is still effective along with the running time gain $\gamma(N, J) > 1$. Moreover, mostly the distance

$$\tilde{d}^* = \rho_{-1}(\mathbf{G}_{\tilde{t}^*}, \tilde{\mathbf{B}}) = \min_{t \in T \subset S} \rho_{-1}(\mathbf{G}_t, \tilde{\mathbf{B}})$$

is lesser than

$$d^{\langle S^* \rangle} = \min_{r^* \in S^* \subset S} \rho_{-1}(\mathbf{G}_{r^*}, \tilde{\mathbf{B}})$$

by

$$S^* = \arg\min_{r \in S} \sum_{j=1}^{J} \xi_j \rho_{-1}(\mathbf{B}_j, \mathbf{G}_r).$$

In ratios, if $\tilde{d}^* < d^{\langle S^* \rangle}$, the inequality

$$\frac{d^{\langle S^* \rangle} - \tilde{d}^*}{d^{\langle S^* \rangle}} > \frac{\tilde{d}_J^* - d_J^{\langle S \rangle}}{\tilde{d}_J^*} \tag{21}$$

is always true. In this way, the inequality (21) means another gain in distances to the aggregation of the experts' rankings: whilst losing in the minimized score (19) against (20) about 1 %, we gain up to 25 % in approximating the Kemeny consensus to the averaged expert ranking.

Discussion

After a lot of modeling operations, the equality

$$\{\mathbf{G}_{\tilde{t}^*}\}_{\tilde{t}^* \in \tilde{T}^*} = \{\mathbf{G}_{r^*}\}_{r^* \in S^*} \tag{22}$$

reveals itself to be true for three objects irrespective of the number of experts. When four objects are ranked, the equality (22) falls out true at 98.5 % rate. The rate decreases by only about 0.3 % for hundreds of experts. By further increment of the number of objects, the likelihood of (22) decreases depending on J weakly. The equality (22) falls out true in every second case when six objects are ranked. For seven objects, it is only every third case.

Cases when $\tilde{d}_J^* > d_J^{\langle S \rangle}$ and $\tilde{d}^* > d^{\langle S^* \rangle}$ occur rarely. A similar sparsity occurs when $|\tilde{T}^*| > |S^*|$. The case with either $\tilde{d}_J^* = d_J^{\langle S \rangle}$ or $\tilde{d}^* = d^{\langle S^* \rangle}$ is pretty frequent. Both equalities are infrequent.

A single Kemeny consensus is determined at one stroke for three objects, statistically, at 75 % rate. When four objects are ranked, this rate decreases down to 36 %, almost twice. And for five objects, only every tenth averaged expert ranking turns out acyclic. So the rate decreases down as the number of objects increases. The number of experts here does not influence as well.

The schemed in Fig. 3 algorithm can be easily adjusted to any subspace of the space (1). This is why, instead of the membership (12), we might gather all the matrices whose distance to the first ranking (7) differs from the averaged expert ranking no more than by 2, 3, etc. However, the running time is increasing then.

Conclusions

The developed algorithm finds a set of Kemeny rankings much faster than the classical straightforward search. Also this set often contains a single Kemeny consensus, what fails by the straightforward search with the weighted Kemeny–Snell distance (2). A single Kemeny consensus is determined at one stroke if the averaged expert ranking turns out acyclic. Thus the problem of selecting a single Kemeny consensus is solved. After $N > 10$, where most known approaches become intractable, the algorithm still is tractable due to searching over only those standard matrices whose distance to the first ranking (7) differs minimally from the distance between this ranking and the averaged expert ranking.

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В.В. Романюк

ШВИДКА УЗГОДЖЕНІСТЬ ЗА КЕМЕНІ НА ОСНОВІ ПОШУКУ ПО СТАНДАРТНИХ МАТРИЦЯХ З МІНІМАЛЬНОЮ ВІДСТАННЮ ДО УСЕРЕДНЕНОГО ЕКСПЕРТНОГО РАНЖУВАННЯ

Проблематика. Розглядається задача ранжування скінченної множини об'єктів.

Мета дослідження. Розробка алгоритму, який дав би змогу пришвидшити пошук узгодженості за Кемені поряд з обґрунтуванням метрики для порівняння ранжувань.

Методика реалізації. Пропонується й обґрунтовується підхід щодо об'єднання експертних ранжувань. Також пропонується й обґрунтовується метрика для порівняння ранжувань.

Результати дослідження. Розроблений алгоритм знаходить множину ранжувань Кемені значно швидше, ніж класичний прямий пошук. Також ця множина часто містить єдину узгодженість за Кемені, що не вдається за прямого пошуку. Крім цього, єдина узгодженість за Кемені визначається відразу, якщо усереднене експертне ранжування виявляється ациклічним. Так розв'язується задача вибору єдиної узгодженості за Кемені.

Висновки. Для 10 і більше об'єктів, де більшість відомих підходів стають незастосовними, алгоритм є реалізовним завдяки пошуку по тільки тих стандартних матрицях, чия відстань до першого ранжування відрізняється від відстані між цим ранжуванням та усередненим експертним ранжуванням на мінімальну величину.

Ключові слова: ранжування; узгодженість за Кемені; усереднене експертне ранжування.

В.В. Романюк

БЫСТРАЯ СОГЛАСОВАННОСТЬ ПО КЕМЕНИ НА ОСНОВЕ ПОИСКА ПО СТАНДАРТНЫМ МАТРИЦАМ С МИНИМАЛЬНЫМ РАССТОЯНИЕМ ДО УСРЕДНЕННОГО ЭКСПЕРТНОГО РАНЖИРОВАНИЯ

Проблематика. Рассматривается задача ранжирования конечного множества объектов.

Цель исследования. Разработка алгоритма, который позволил бы ускорить поиск согласованности по Кемени вместе с обоснованием метрики для сравнения ранжирований.

Методика реализации. Предлагается и обосновывается подход относительно объединения экспертных ранжирований. Также предлагается и обосновывается метрика для сравнения ранжирований.

Результаты исследования. Разработанный алгоритм находит множество ранжирований Кемени гораздо быстрее, чем классический прямой поиск. Также это множество часто содержит единственную согласованность по Кемени, что не удается при прямом поиске. Кроме этого, единственная согласованность по Кемени определяется сразу, если усредненное экспертное ранжирование оказывается ациклическим. Так решается задача выбора единственной согласованности по Кемени.

Выводы. Для 10 и более объектов, где большинство известных подходов становятся неисполнимыми, алгоритм является осуществимым благодаря поиску по только тем стандартным матрицам, чье расстояние к первому ранжированию отличается от расстояния между этим ранжированием и усредненным экспертным ранжированием на минимальную величину.

Ключевые слова: ранжирование; согласованность по Кемени; усредненное экспертное ранжирование.

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