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OPERATORS OF STOCHASTIC DIFFERENTIATION ON SPACES OF REGULAR TEST AND GENERALIZED FUNCTIONS IN THE LÉVY WHITE NOISE ANALYSIS

The operators of stochastic differentiation, which are closely related with stochastic integrals and with the Hida stochastic derivative, play an important role in the classical white noise analysis. In particular, one can use these operators in order to study properties of solutions of normally ordered stochastic equations, and properties of the extended Skorohod stochastic integral. So, it is natural to introduce and to study analogs of the mentioned operators in the Lévy white noise analysis. In this paper, using the theory of Hilbert equipments, in terms of the Lytvynov's generalization of the chaotic representation property we introduce operators of stochastic differentiation on spaces from parametrized regular rigging of the space of square integrable with respect to the measure of a Lévy white noise functions. Then we establish some properties of introduced operators. This gives a possibility to extend to the Lévy white noise analysis and to deepen the well-known results of the classical white noise analysis that are connected with the operators of stochastic differentiation.

Keywords: operator of stochastic differentiation, extended stochastic integral, Hida stochastic derivative, Lévy process.

Introduction

Denote $R_+ := [0,+\infty)$. Let $L = (L_t)_{t\in R_+}$ be a Lévy process, i.e., a random process on R_+ with stationary independent increments and such that $L_0 = 0$ (see, e.g., [1] for more details). In particular cases, when L is a Wiener or Poisson process, any square integrable random variable can be decomposed in a series of repeated Itô stochastic integrals from nonrandom functions with respect to L. This property of L, known as the *chaotic representation property* (CRP) (see, e.g., [2] for details), plays a very important role in the stochastic analysis. In particular, it can be used in order to construct the extended Skorohod stochastic integral (e.g., [3]) and its adjoint operator – the Hida stochastic derivative. Unfortunately, for a general Lévy process the CRP does not hold (e.g., [4]).

There are different generalizations of the CRP for Lévy processes (see, e.g., [5–8]). In this paper we deal with the Lytvynov's generalized CRP [5] that is based on orthogonalization of continuous polynomials in the space of square integrable random variables (L^2). Using this generalized CRP, one can construct the extended Skorohod stochastic integral with respect to L and the Hida stochastic derivative on the mentioned space [8] and on spaces of test and generalized functions from its riggings [9–11]. Together with the mentioned operators, it is natural to introduce and to study so-called *operators of stochastic differentiation*, by analogy with the classical white noise analysis [12, 13], the "Gamma-analysis" [14, 15] and the "Meixner analysis" [16, 17]. These operators are closely related with the extended stochastic integral with respect to a Lévy process and with the corresponding Hida stochastic derivative, and, by analogy with the "classical case", can be used, in particular, in order to study properties of solutions of normally ordered stochastic equations, and properties of the extended Skorohod stochastic integral. In this paper we introduce such operators on spaces of the so-called regular parametrized rigging of (L^2) ([10, 11]) and establish some properties of the mentioned operators.

Problem definition

The aim of this paper is to introduce the operators of stochastic differentiation on spaces of a parameterized regular rigging of the space of square integrable random variables; and to establish some properties of these operators.

Preliminaries

In this paper we deal with a real-valued locally square integrable Lévy process L without Gaussian part and drift. As is well known, the characteristic function of such a process is

$$E[e^{iuL_t}] = \exp[t\int_R (e^{iux} - 1 - iux)\nu(dx)], \qquad (1)$$

where ν is the Lévy measure of L, E denotes the expectation. We assume that ν *is a Radon measure whose support contains an infinite number* *of points,* $\nu(\{0\})=0$, *there exists* $\varepsilon>0$ *such that* $\int_R x^2 e^{\varepsilon|x|}\nu(dx)<\infty$, *and* $\int_R x^2\nu(dx)=1$.

Let us define a measure of the white noise of L. Let D denote the set of all real-valued infinite-differentiable functions on R_+ with compact supports. As is well known, D can be endowed by the projective limit topology generated by some Sobolev spaces (see, e.g., [18]}). Let D' be the set of linear continuous functionals on D. For $\omega\in D'$ and $\varphi\in D$ denote $\langle\omega,\varphi\rangle:=\omega(\varphi)$; note that one can understand $\langle\cdot,\cdot\rangle$ as the dual pairing generated by the scalar product in the space $L^2(R_+)$ of square integrable with respect to the Lebesgue measure real-valued functions on R_+. The notation $\langle\cdot,\cdot\rangle$ will be preserved for dual pairings in tensor powers of spaces.

A probability measure μ on $(D',C(D'))$, where C denotes the cylindrical σ-algebra, with the Fourier transform

$$\int_{D'} e^{i\langle\omega,\varphi\rangle}\mu(d\omega)$$

$$=\exp[\int_{R_+\times R}(e^{i\varphi(u)x}-1-i\varphi(u)x)du\nu(dx)],\ \varphi\in D, \quad (2)$$

is called the *Lévy white noise measure* (e.g., [5]).

Denote $(L^2):=L^2(D',C(D'),\mu)$ the space of real-valued square integrable with respect to μ functions on D'; let also $H:=L^2(R_+)$. Let $f\in H$ and a sequence $(\varphi_k\in D)_{k\in N}$ converge to f in H as $k\to\infty$. One can show ([5, 8, 19]) that $\langle\circ,f\rangle:=(L^2)-\lim_{k\to\infty}\langle\circ,\varphi_k\rangle$ is well defined as an element of (L^2). Let us consider $\langle\circ,1_{[0,t)}\rangle\in(L^2)$, $t\in R_+$ (here and below 1_A denotes the indicator of a set A). It follows from (1) and (2) that $(\langle\circ,1_{[0,t)}\rangle)_{t\in R_+}$ can be identified with a Lévy process L.

Consider the Lytvynov's generalization of the CRP (see [5] for details). Denote by $\hat{\otimes}$ the symmetric tensor product. For $n\in Z_+:=N\cup\{0\}$ set

$$P_n:=\left\{\varphi(x)=\sum_{k=0}^{m}\langle x^{\otimes k},\varphi^{(k)}\rangle \mid x\in D',\varphi^{(k)}\in D^{\hat{\otimes}k},m\le n\right\}.$$

Denote by $\bar{P}_n$ the closure of P_n in (L^2). Let for $n\in N$ P_n be defined from the condition $\bar{P}_n=\mathrm{P}_n\oplus\bar{P}_{n-1}$, $\mathrm{P}_0:=\bar{P}_0$. Let $f^{(n)}\in D^{\hat{\otimes}n}$, $n\in Z_+$. Denote by $:\langle\circ^{\otimes n},f^{(n)}\rangle:$ the orthogonal projection of a monomial $\langle\circ^{\otimes n},f^{(n)}\rangle$ onto P_n. Let us define scalar products $(\cdot,\cdot)_{\text{ext}}$ on $D^{\hat{\otimes}n}$, $n\in Z_+$ $(D^{\hat{\otimes}0}:=R)$, by setting for $f^{(n)},g^{(n)}\in D^{\hat{\otimes}n}$

$$(f^{(n)},g^{(n)})_{\text{ext}}$$

$$:=\frac{1}{n!}\int_{D'}:\langle\omega^{\otimes n},f^{(n)}\rangle::\langle\omega^{\otimes n},g^{(n)}\rangle:\mu(d\omega),$$

and let $|\cdot|_{\text{ext}}$ be the corresponding norms. Denote by $H_{\text{ext}}^{(n)}$, $n\in Z_+$, the completions of $D^{\hat{\otimes}n}$ with respect to the norms $|\cdot|_{\text{ext}}$. For $F^{(n)}\in H_{\text{ext}}^{(n)}$ we define a Wick monomial $:\langle\circ^{\otimes n},F^{(n)}\rangle:\overset{\text{def}}{=}(L^2)-\lim_{k\to\infty}:\langle\circ^{\otimes n},f_k^{(n)}\rangle:$, here for each $k\in N$ $f_k^{(n)}\in D^{\hat{\otimes}n}$ and $f_k^{(n)}\to F^{(n)}$ as $k\to\infty$ in $H_{\text{ext}}^{(n)}$ (well-posedness of this definition can be proved by the method of "mixed sequences"). One can show ([5]) that $F\in(L^2)$ if and only if there exists a unique sequence of kernels $F^{(n)}\in H_{\text{ext}}^{(n)}$, $n\in Z_+$, such that

$$F=\sum_{n=0}^{\infty}:\langle\circ^{\otimes n},F^{(n)}\rangle: \quad (3)$$

with

$$E|F|^2=\|F\|_{(L^2)}^2=\int_{D'}|F(\omega)|^2\mu(d\omega)=\sum_{n=0}^{\infty}n!|F^{(n)}|_{\text{ext}}^2<\infty.$$

Note that $H_{\text{ext}}^{(1)}=H$ and for $n\in N\setminus\{1\}$ one can identify $H^{\hat{\otimes}n}$ with the proper subspace of $H_{\text{ext}}^{(n)}$ that consists of "vanishing on diagonals" elements ([5, 8]). In this sense the *space* $H_{\text{ext}}^{(n)}$ *is an extension of* $H^{\hat{\otimes}n}$.

For $\beta\in(0,1]$ and $q\in Z$ or $\beta=0$ and $q\in Z_+$ define $(L^2)_q^\beta\subseteq(L^2)$ as a Hilbert space that consists of elements of form (3) for which

$$\|F\|_{(L^2)_q^\beta}^2=\sum_{n=0}^{\infty}(n!)^{1+\beta}2^{qn}|F^{(n)}|_{\text{ext}}^2<\infty. \quad (4)$$

Let $(L^2)_{-q}^{-\beta}$ be the Hilbert space of generalized functions that is dual of $(L^2)_q^\beta$ with respect to

(L^2), i.e., $(L^2)^{-\beta}_{-q}$ is the negative space of the chain

$$(L^2)^{\beta}_{q} \subseteq (L^2) \subseteq (L^2)^{-\beta}_{-q}. \qquad (5)$$

This space consists of formal series of form (3) with $\| F\|^2_{(L^2)^{-\beta}_{-q}} = \sum_{n=0}^{\infty}(n!)^{1-\beta}2^{-qn}|F^{(n)}|^2_{\text{ext}} < \infty$ (cf. (4)). The chain constructed above is called a *parameterized regular rigging of the space* (L^2). In what follows, we denote all spaces from this chain by $(L^2)^{\beta}_{q}$, $\beta \in [-1,1]$, $q \in Z$ (these assumptions about β and q will be accepted on default). The norms in these spaces are given, obviously, by formula (4).

Recall the construction of the extended stochastic integral with respect to L. Let $F \in (L^2)^{\beta}_{q} \otimes H$. Then, obviously, F can be uniquely presented in the form

$$F(\cdot) = \sum_{n=0}^{\infty} :\langle \circ^{\otimes n}, F^{(n)}_{\cdot}\rangle:, \; F^{(n)}_{\cdot} \in H^{(n)}_{\text{ext}} \otimes H.$$

Let $t_1, t_2 \in [0, +\infty]$, $t_1 < t_2$ (we accept this on default). For the kernel $F^{(n)}_{\cdot}$, $n \in N$, we select a representative (a function) $\dot{f}^{(n)}_{\cdot} \in F^{(n)}_{\cdot}$ such that $\dot{f}^{(n)}_{u}(u_1,\dots,u_n) = 0$ if for some $k \in \{1,\dots,n\}$ $u = u_k$. Let $\tilde{f}^{(n)}_{[t_1,t_2)}$ be the symmetrization of $\dot{f}^{(n)}_{\cdot} 1_{[t_1,t_2)}(\cdot)$ by $n+1$ variables. Define $\hat{F}^{(n)}_{[t_1,t_2)} \in H^{(n+1)}_{\text{ext}}$ as an equivalence class in $H^{(n+1)}_{\text{ext}}$ that is generated by $\tilde{f}^{(n)}_{[t_1,t_2)}$. It is proved in [8] that $\hat{F}^{(n)}_{[t_1,t_2)}$ is well defined (in particular, $\hat{F}^{(n)}_{[t_1,t_2)}$ does not depend on the choice of $\dot{f}^{(n)}_{\cdot}$) and $|\hat{F}^{(n)}_{[t_1,t_2)}|_{\text{ext}} \le |F^{(n)}_{\cdot}|_{H^{(n)}_{\text{ext}} \otimes H}$.

Definition. We define the extended stochastic integral $\int_{t_1}^{t_2} \circ(u)\hat{d}L_u : (L^2)^{\beta}_{q} \otimes H \to (L^2)^{\beta}_{q-1}$ by the formula

$$\int_{t_1}^{t_2} F(u)\hat{d}L_u := \sum_{n=0}^{\infty} :\langle \circ^{\otimes n+1}, \hat{F}^{(n)}_{[t_1,t_2)}\rangle:, \qquad (6)$$

here $\hat{F}^{(0)}_{[t_1,t_2)} := F^{(0)}_{\cdot} 1_{[t_1,t_2)}(\cdot)$.

As it is shown in [10, 11], this integral is a linear *continuous* operator. Note that sometimes it can be convenient to define the stochastic integral by formula (6) as a linear operator $\int_{t_1}^{t_2} \circ(u)\hat{d}L_u : (L^2)^{\beta}_{q} \otimes H \to (L^2)^{\beta}_{q}$. If $\beta = -1$ then this operator is still continuous one [10], for $\beta \in (-1,1]$ this is not the case. But if we accept a set $\{F \in (L^2)^{\beta}_{q} \otimes H : \| \int_{t_1}^{t_2} F(u)\hat{d}L_u \|_{(L^2)^{\beta}_{q}} < \infty\}$ as the domain of this integral then the last will be a *closed* operator [10, 11]. If F is integrable by Itô then F is integrable in the extended sense and the integrals coincide [8]. Finally, the operator $1_{[t_1,t_2)}(\cdot)\partial_{\cdot}$, which is adjoint to the extended stochastic integral, is called the *Hida stochastic derivative* (in the case $t_1 = 0$ and $t_2 = +\infty$ we denote this operator by $\partial_{\cdot}$). The reader can find a detailed information about construction of this operator in terms of the Lytvynov's CRP and about its properties in, e.g., [8–11].

Operators of stochastic differentiation

In order to define operators of stochastic differentiation, we need a preparation. Let $F^{(m)} \in H^{(m)}_{\text{ext}}$, $G^{(n)} \in H^{(n)}_{\text{ext}}$, $m,n \in Z_+$. We select representatives (functions) $\dot{f}^{(m)} \in F^{(m)}$, $\dot{g}^{(n)} \in G^{(n)}$ from the equivalence classes $F^{(m)}, G^{(n)}$, and set $d(\dot{f}^{(m)}\dot{g}^{(n)}) \times (u_1,\dots,u_m; u_{m+1},\dots,u_{m+n}) = \dot{f}^{(m)}(u_1,\dots,u_m)\dot{g}^{(n)}(u_{m+1},\dots,u_{m+n})$ if for all $i \in \{1,\dots,m\}$ and $j \in \{m+1,\dots,m+n\}$ $u_i \ne u_j$, and $d(\dot{f}^{(m)}\dot{g}^{(n)})(u_1,\dots,u_{m+n}) = 0$ in other cases. Let $s(\dot{f}^{(m)}\dot{g}^{(n)})$ be the symmetrization of $d(\dot{f}^{(m)}\dot{g}^{(n)})$ by all variables, $F^{(m)} \Diamond G^{(n)} \in H^{(m+n)}_{\text{ext}}$ be an equivalence class in $H^{(m+n)}_{\text{ext}}$ that is generated by $s(\dot{f}^{(m)}\dot{g}^{(n)})$ (i.e., $s(\dot{f}^{(m)}\dot{g}^{(n)}) \in F^{(m)} \Diamond G^{(n)}$).

Lemma. The element $F^{(m)} \Diamond G^{(n)} \in H^{(m+n)}_{\text{ext}}$ is well defined (in particular, $F^{(m)} \Diamond G^{(n)}$ does not depend on the choice of representatives of $F^{(m)}$ and $G^{(n)}$) and

$$|F^{(m)} \Diamond G^{(n)}|_{\text{ext}} \le |F^{(m)}|_{\text{ext}} |G^{(n)}|_{\text{ext}}. \qquad (7)$$

The proof is similar to the proof of Lemma 3.1 in [20], therefore we shall confine ourselves to the short description of main steps. First, using the explicit formula for $|\cdot|_{\text{ext}}$ [5, 8, 10], by direct calculation one can show that $|s(\dot{f}^{(m)}\dot{g}^{(n)})|_{\text{ext}} \le$

$\leq |\dot{f}^{(m)}|_{\text{ext}} |\dot{g}^{(n)}|_{\text{ext}}$, therefore there exists an equivalence class $F^{(m)} \Diamond G^{(n)} \in H_{\text{ext}}^{(m+n)}$ such that $s(\dot{f}^{(m)} \dot{g}^{(n)}) \in F^{(m)} \Diamond G^{(n)}$ and estimate (7) is fulfilled. Second, using properties of the operation $s(\circ)$ and the obtained estimate for the norms $|\cdot|_{\text{ext}}$ of functions of a form $s(\dot{f}^{(m)} \dot{g}^{(n)})$, one can show that the equivalence class $F^{(m)} \Diamond G^{(n)} \in H_{\text{ext}}^{(m+n)}$ does not depend on the choice of representatives from $F^{(m)}$ and $G^{(n)}$, this completes the proof.

Further, for $F^{(m)} \in H_{\text{ext}}^{(m)}$ and $f^{(n)} \in H_{\text{ext}}^{(n)}$, $m, n \in Z_+$, $m > n$, we define a generalized scalar product $(F^{(m)}, f^{(n)})_{\text{ext}} \in H_{\text{ext}}^{(m-n)}$ by setting for each $g^{(m-n)} \in H_{\text{ext}}^{(m-n)}$ $((F^{(m)}, f^{(n)})_{\text{ext}}, g^{(m-n)})_{\text{ext}} = (F^{(m)}, g^{(m-n)} \Diamond f^{(n)})_{\text{ext}}$. As is easy to verify by the Cauchy–Bunyakovsky inequality and (7), this definition is correct and

$$|(F^{(m)}, f^{(n)})_{\text{ext}}|_{\text{ext}} \leq |F^{(m)}|_{\text{ext}} |f^{(n)}|_{\text{ext}}. \qquad (8)$$

Definition. Let $f^{(n)} \in H_{\text{ext}}^{(n)}$, $n \in N$. We define the operator of stochastic differentiation

$$(D^n \circ)(f^{(n)}) : (L^2)_q^{\beta} \to (L^2)_{q-1}^{\beta}$$

by the formula

$$(D^n F)(f^{(n)}) := \sum_{m=0}^{\infty} \frac{(m+n)!}{m!} :\langle \circ^{\otimes m}, (F^{(m+n)}, f^{(n)})_{\text{ext}} \rangle :, \qquad (9)$$

where $F^{(m)} \in H_{\text{ext}}^{(m)}$ are the kernels from decomposition (3) for $F \in (L^2)_q^{\beta}$. Also we denote

$$(D\circ)(f^{(1)}) := (D^1 \circ)(f^{(1)}).$$

By direct calculation with use (3) and (8) one obtains the estimate

$$\|(D^n F)(f^{(n)})\|^2_{(L^2)^{\beta}_{q-1}} \leq 2^{-qn} \times |f^{(n)}|^2_{\text{ext}} \max_{m \in Z_+} \left[\left(\frac{(m+n)!}{m!} \right)^{1-\beta} 2^{-m} \right] \|F\|^2_{(L^2)^{\beta}_q},$$

whence it follows that $(D^n \circ)(f^{(n)})$ is a well defined linear continuous operator.

Consider properties of $(D^n \circ)(f^{(n)})$.

Theorem. 1) For $g_1^{(1)}, \ldots, g_n^{(1)} \in H_{\text{ext}}^{(1)} = H$

$$(D(\cdots(D((D\circ)(g_1^{(1)})))(g_2^{(1)}) \cdots))(g_n^{(1)}) = (D^n \circ)(g_1^{(1)} \Diamond \cdots \Diamond g_n^{(1)}).$$

2) For each $F \in (L^2)_q^{\beta}$ the kernels $F^{(m)} \in H_{\text{ext}}^{(m)}$ from decomposition (3) can be presented in the form

$$F^{(m)} = \frac{1}{m!} E(D^m F),$$

i.e., for each $f^{(m)} \in H_{\text{ext}}^{(m)}$

$$(F^{(m)}, f^{(m)})_{\text{ext}} = \frac{1}{m!} E((D^m F)(f^{(m)})),$$

here $E\circ = \langle\langle \circ, 1 \rangle\rangle$ is a generalized expectation, $\langle\langle \circ, \circ \rangle\rangle$ denotes the dual pairing in chain (5).

3) The adjoin to D^n operator has a form

$$(D^n G)(f^{(n)})^* = \sum_{m=0}^{\infty} :\langle \circ^{\otimes m+n}, G^{(m)} \Diamond f^{(n)} \rangle :\in (L^2)_{-q}^{-\beta},$$

where $G \in (L^2)_{1-q}^{-\beta}$, $f^{(n)} \in H_{\text{ext}}^{(n)}$, and $G^{(m)} \in H_{\text{ext}}^{(m)}$ are the kernels from decomposition (3) for G.

4) For all $G \in (L^2)_{1-q}^{-\beta}$ and $f^{(1)} \in H_{\text{ext}}^{(1)} = H$

$$(DG)(f^{(1)})^* = \int_{R_+} G \cdot f^{(1)}(u) \hat{d} L_u \in (L^2)_{-q}^{-\beta}.$$

5) For all $F \in (L^2)_q^{\beta}$ and $f^{(1)} \in H_{\text{ext}}^{(1)}$

$$(DF)(f^{(1)}) = \int_{R_+} \partial_u F \cdot f^{(1)}(u) \, du \in (L^2)_{q-1}^{\beta},$$

here the integral in the right hand side is a Pettis one (the weak integral). Formally $\partial_{\cdot} \circ = (D\circ)(\delta_{\cdot})$, where $\delta_{\cdot}$ is the Dirac delta-function.

6) Let $F \in (L^2)_q^{\beta} \otimes H$, $f^{(1)} \in H_{\text{ext}}^{(1)}$. Then

$$(D\int_{t_1}^{t_2} F(u) \hat{d} L_u)(f^{(1)}) = \int_{t_1}^{t_2} (DF(u))(f^{(1)}) \hat{d} L_u + \int_{t_1}^{t_2} F(u) f^{(1)}(u) \, du \in (L^2)_{q-1}^{\beta},$$

here the last integral is a Pettis one.

The proof is quite analogous to the proofs of the corresponding statements in [15, 16] and consists in direct calculations with use, in particular, estimates (7) and (8).

Finally we note that sometimes it can be convenient to consider the operators of stochastic differentiation, given by formula (9), as ones acting in the space $(L^2)^{\beta}_{q}$. By analogy with [16] one can show that for $\beta = 1$ such operators are still continuous ones, for $\beta \in [-1,1)$ they can be defined as *closed* ones. The properties described above hold true up to obvious modifications.

Conclusions

In this paper the operators of stochastic differentiation are introduced on spaces from parametrized regular rigging of the space (L^2) of square integrable with respect to the measure of a Lévy white noise functions; and some properties of these operators are established. This gives a possibility, in particular, to study properties of the extended stochastic integral and of solutions of normally ordered stochastic equations.

In the forthcoming papers we'll consider elements of the Wick calculus and its interconnection with the stochastic differentiation and integration, operators of stochastic differentiation on spaces of another (nonregular) rigging of (L^2), etc.

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