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FLOWS FEATURES IN THE INTERNAL CHANNELS OF INVOLUTE EXTERNAL GEAR PUMPS

Results of investigations of external gear pump with involute tooth profile which goal was to obtain the flow patterns in the suction and discharge chambers, and in meshing region and examining the effect on them of operating parameters of the pump were presented. Flow patterns were obtained by visualisation of fluid flow with a high-speed video recording of working process of the pump with a lid made of a transparent material. As result of video analysis appearance of cavities in the fluid closed in trapped volume was found. Process of the emergence of a cavitation bubble, its growth and collapse was reviewed. The calculation dependencies that link up dimensions of cavitation bubbles and the pressure in the surrounding liquid were shown. The change of the trapped volume dimensions for given pump and type of pressure changes inside were determined. Using π-theorem dependencies in dimensionless form were obtained, criterion analysis of the results was performed showing the dependence of the pressure in the trapped volume from a number of complexes, such as centrifugal Reynolds number. It was revealed that in the trapped volume compression of fluid and cavitation occur; intensity depends on the operating conditions of the pump.

Keywords: gear pump, flow visualisation, trapped volume, cavitation.

Introduction

External gear pumps are the one of the most common sources of hydraulic power in the hydraulic systems due to their relative simplicity, reliability and low requirements on the purity of the working fluid. During operation of external gear pump trapped volume appears between its working bodies formed by the surfaces of the gear teeth and the surfaces of the side covers, which value varies leading to formation of high pressure region. Since liquids have a low coefficient of compressibility, the pressure can reach high values (order greater than rated pressure of the pump). Compression of fluid contained in the cavity can lead to deformation of the pump, which is detrimental to its longevity and energy characteristic. This issue is discussed in several papers [1–5]. One of the ways to study the working process in gear pumps is the filming of running pump by high speed camera. These studies were conducted at Wroclaw University of Technology.

Problem statement

The purpose of this study is to obtain a qualitative flow patterns in specific areas of external gear pump, analysis and study influence of the conditions at the inlet and outlet of the pump and the speed gears on the flow pattern.

Results of the investigation

In external involute gear pump there are following specific areas: input channel, inlet chamber, intertooth space, outlet chamber, outlet channel (Fig. 1).

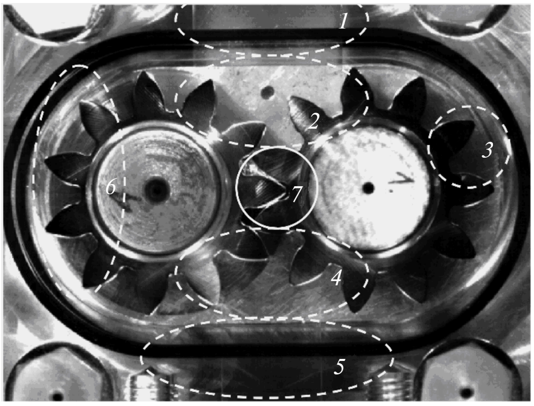


Fig. 1. Typical regions in the external involute gear pump: *1* – suction channel, *2* – suction chamber, *3* – intertooth space, *4* – pressure chamber, *5* – pressure port, *6* – inlet bridge, *7* – outlet bridge (the bridge – region located between zones of high and low pressure)

Investigation of flows in the inner cavity of a gear pump was carried out by shooting running pump with high-speed camera Phantom v7.3 at speeds up to 10,000 frames per second with a resolution of 640×480 pixels. The equipment enables to shoot at up to 190,467 frames per second in standard and up to 500,000 fps in turbo mode. However, if the frame rate increases it leads to reducing resolution of the apparatus. Analyzed pump is the analogue of real pump, difference is that for visual observation of flows its body was made of plexiglass (Fig. 2).

Hydraulic schematic diagram of the experimental setup is shown in Fig. 3. It comprises the monitoring pump *1* actuated by a motor, which allows to set the speed of the driving gear of pump, hydraulic tank *2*, adjustable throttle *3*, purposed to create reduced pressure at the pump inlet, pressure transducers *4* and *14* on the inlet and outlet, an additional pump *5* to pressurize pump inlet, flowmeter *6*, temperature sensors *7* and *13* on the discharge and suction line, adjustable throttle *8* to set the pressure in the pump discharge line, an additional pump unit *9*, filter *10*, cooler *11* and heater *12*.

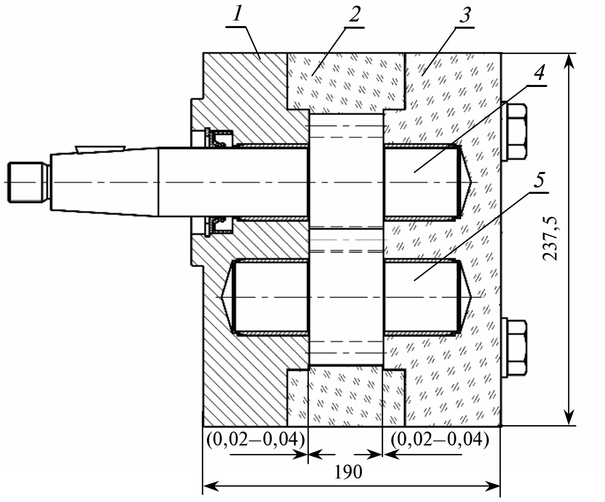


Fig. 2. Construction of the gear pump used in the experiment: *1* – front cover (metal), *2* – middle part (plexiglass), *3* – back cover (plexiglass), *4* – pinion gear, *5* – driven gear

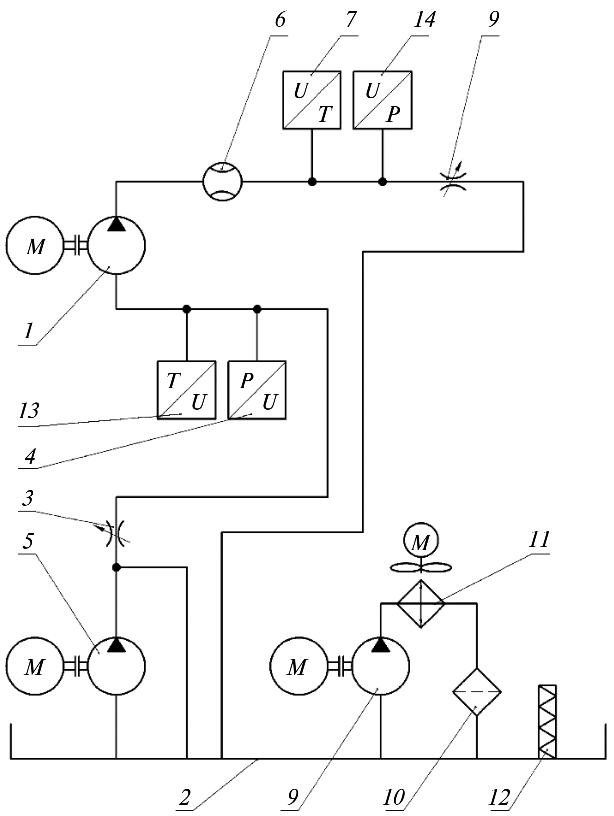


Fig. 3. Schematic diagram of the experimental setup

General view of the experimental setup is shown in Fig. 4.

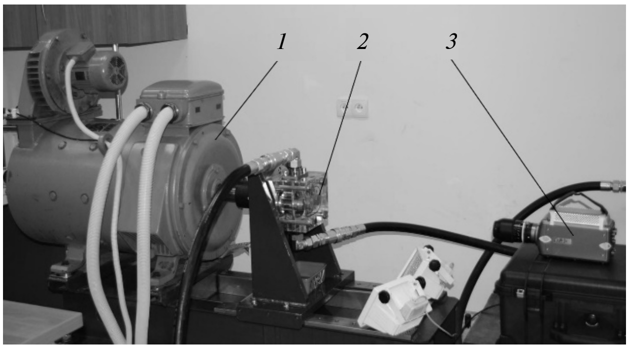


Fig. 4. General view of the experimental setup: *1* – motor, *2* – investigated pump, *3* – high-speed video camera

Studies were performed for the following modes of operation of the pump: speed gears – 500 and 1000 rpm, pressure in the suction chamber –0,05, 0 and 0,05 MPa, pressure in the pumping chamber of 2,5 and 3,5 MPa. Working fluid HLP 68, temperature of fluid 50 °C.

During operation of the pump lowering pressure in the intertooth space, occurrence and further collapse of bubbles can be observed. This occurs when the gear teeth goes out of the gearing. Trapped volume which value is increasing leading to decreasing the pressure in it forms there. Fig. 5, *b* shows the beginning of bubble formation, its growth (Fig. 5, *c*, *d*) and collapse (Fig. 5, *e*, *f*) when the trapped volume disappears.

Considered flow pattern characterizes all features of cavitation, i.e. formation in dropping liquid cavities filled with steam, gas, or a mixture thereof. Cavitation bubbles are formed in locations where the fluid pressure drops below a certain critical value (for the real fluid approximately equal to the saturated vapor pressure of the liquid at a given temperature).

Conditions resulting from the collapse of bubbles (a pressure of about 10^3 MPa and a temperature of 10^4 °C) can lead to the destruction of the gears machine and the deterioration of its characteristics. In this case, cavitation occurs in the liquid that closed in trapped volume, which volume increases leading to a pressure drop therein.

The pressure of the liquid around the bubble can be determined by the formula [6]

$$p_{\text{med}} = p_0 \frac{R_0^3}{R^3} + p_s \left(1 - \frac{R_0^3}{R^3}\right) + 2\sigma \left(\frac{R_0^2}{R^3} - \frac{1}{R}\right),$$

where R – radius of the bubble, p_s – the saturated vapor pressure. Index 0 stands for values corresponding to the initial value of the development of the bubble; σ – surface tension coefficient.

Basing on this formula the critical bubble radius and pressure of the medium surrounding the bubble can be found:

$$R_{\text{crit}} = \sqrt{3} R_0 \sqrt{\frac{R_0}{2\sigma} \cdot \left(p_0 - p_{\text{s}} + \frac{2\sigma}{R_0} \right)} =$$

$$= \sqrt{3} R_0 \sqrt{\frac{R_0}{2\sigma} p_{r0}} = \frac{4\sigma}{3(p_{\text{crit}} - p_{\text{s}})},$$

$$(p_{\text{med}})_{\text{cirt}} = -\frac{2}{3\sqrt{3}} \left(\frac{2\sigma}{R_0} \right) \sqrt{\frac{2\sigma}{R_0} \frac{1}{p_{r0}}} = p_{\text{s}} \frac{2}{3} \frac{2\sigma}{R_{\text{cirt}}}.$$

As it can be seen from the above formulas forces of surface tension σ play an important role.

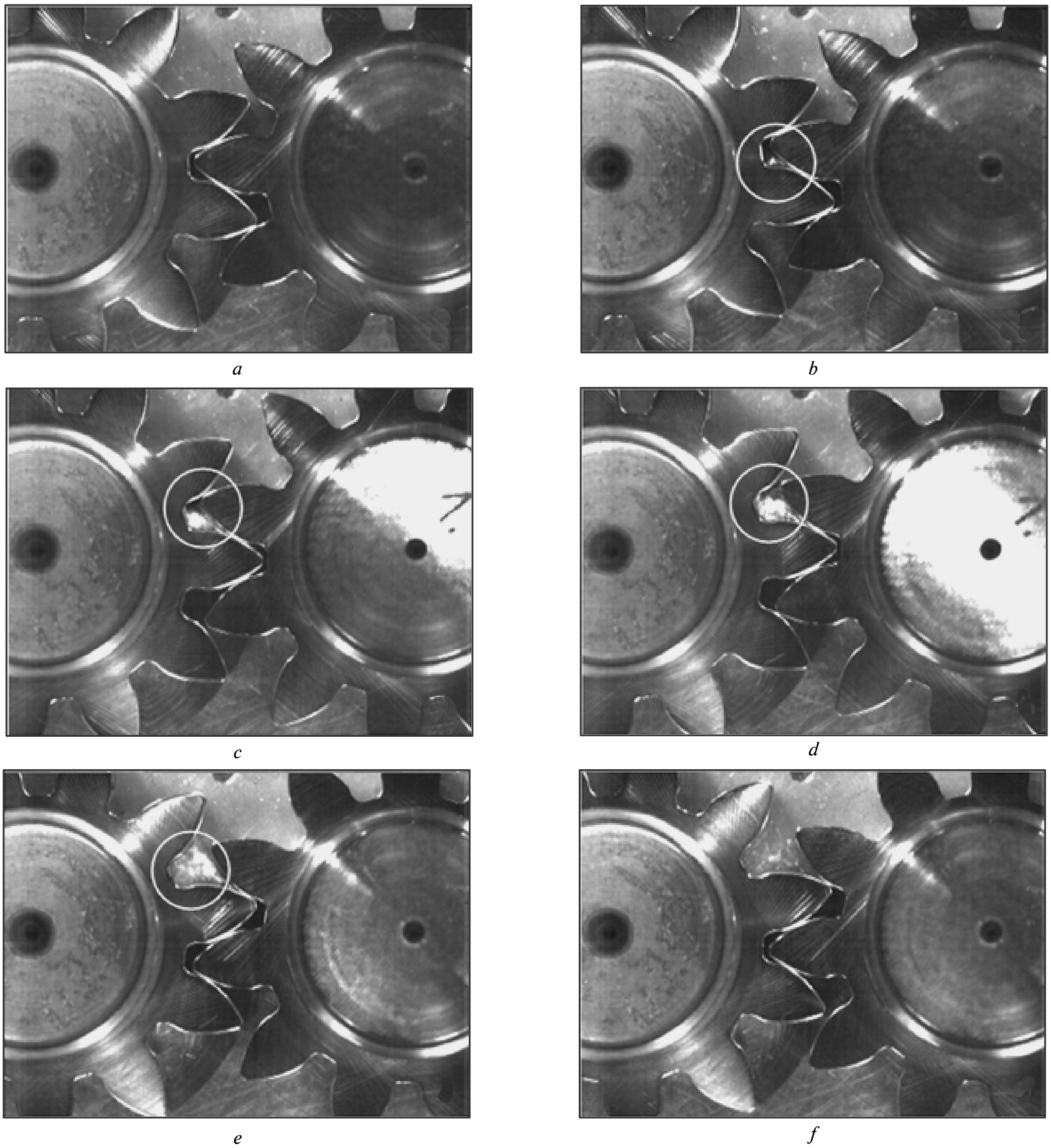


Fig. 5. The process of formation of the cavitation zone in the trapped volume that formed by engaged teeth. Suction chamber shown in the upper part of the figures, the discharge chamber – at the bottom (rotation speed of 500 rpm, inlet pressure 0 MPa, outlet pressure 2,5 MPa)

In order to observe the deformation of the bubble in ideal and viscous fluids, it is necessary on the basis of the equations of motion to formulate the condition of the radius of the bubble, depending on the surrounding factors. For an ideal fluid this equation can be written as:

$$R\frac{d^2R}{dt^2}+\frac{3}{2}\left(\frac{dR}{dt}\right)^2=\frac{p_0+p_s}{\rho}, \quad (1)$$

where ρ — fluid density.

The solution of equation (1) can be obtained at the following initial conditions: $t=0$, $R=R_0$ and $\left(\frac{dR}{dt}\right)_{t=0}=0$. Then for the first and second derivatives we obtain:

$$\left(\frac{dR}{dt}\right)^2=\frac{2}{3}\frac{p+p_s}{\rho}\left(1-\frac{R_0^3}{R^3}\right),$$

$$\left(\frac{d^2R}{dt^2}\right)^2=\frac{p+p_s}{\rho}\frac{R_0^3}{R^4}.$$

The obtained results indicate a rapid change in velocity in the initial period of motion and expanding of sphere. In the case where the liquid, viscous, on the basis of the equations can be obtained:

$$\frac{p-p_0}{\rho}=\frac{1}{r}\left[R^2\frac{d^2R}{dt^2}+2R\left(\frac{dR}{dt}\right)^2\right]\frac{1}{2}\frac{R^4}{r^4}\left(\frac{dR}{dt}\right)^2. \quad (2)$$

Exclusion of members describing the viscosity of the considered liquid is the peculiarity of this equation. However, this conclusion can not be done because the effect of viscosity and its accounting in the corresponding equations associated with boundary conditions.

For a Newtonian fluid accounting viscosity, equation (2) describing the change in bubble radius can be written:

$$\frac{p-p_0}{\rho}=R\frac{d^2R}{dt^2}+\frac{3}{2}\left(\frac{dR}{dt}\right)^2+\frac{4\mu}{\rho R}\frac{dR}{dt}.$$

To determine the pressure in the trapped volume it is necessary to know the value of changes in the trapped volume during rotation of gears. In the external involute gear pump trajectory of tooth contact is a straight line called the line of action, and the segment of line in which the contact pairs of teeth — the active part line of action that defines the beginning and end of the gearing pairs of conjugate teeth.

After determination of the dependence of the trapped volume, depending on the coordinates of the contact of teeth point in the line of action, the pressure increase in the area of existence trapped volume can be calculated.

Using the methods [5] of defining a point on the line of action, in which the TV (trapped volume) takes the minimum value

$$x_{\min}=r_0\,\mathrm{tg}\alpha-0{,}5t_0,$$

where r_0 — radius of the base circle; t_0 — circular pitch; α — pressure angle.

Coordinate of a point x_0 on action line corresponding to the moment of formation of TV can be found from the following equation:

$$x_0=r_0(2\,\mathrm{tg}\alpha-\mathrm{tg}\gamma_e),$$

where γ_e — angle of the top of gears involute.

Further, location of the point of action of the second pair of gear teeth on line of action when coming out of action the first pair of teeth can be defined. The coordinates of the point x_k can be determined according to the following equation:

$$x_k=r_0\,\mathrm{tg}\gamma_e-t_0.$$

For gears with the same number of teeth the dependence of the square of the TV from location of point action of gears on line of action can be written as:

$$S_{TV}=\frac{t_0}{3r_0}\times$$

$$\times\begin{bmatrix}3x^2+3(t_0-2r_0\mathrm{tg}\alpha)x+t_0^2+\\+6r_0^2\mathrm{tg}^2\alpha-3r_0t_0\mathrm{tg}\alpha+3r_0^2-3R_i^2\end{bmatrix}-2S_z,$$

where x — coordinate of a point on the line of action.

The area of the tooth S_z was determined by the formula

$$S_z=r_0^2\frac{\mathrm{tg}^3(\gamma_e)}{3}+$$

$$+R_e^2\left(\frac{\varphi}{2}+\mathrm{inv}\alpha+\mathrm{inv}\gamma_e\right)-R_i^2\left(\frac{\varphi}{2}+\mathrm{inv}\alpha\right),$$

where φ — angle of the arc of tooth on pitch circle; $\mathrm{inv}\alpha$ and $\mathrm{inv}\gamma$ functions and are defined by $\mathrm{inv}x=\mathrm{tg}x-x$.

Increase of pressure in TV can be determined from the following equation:

$$\Delta p = \frac{\Delta V}{V_{\text{TV}}} \cdot E ,$$

where ΔV – change of TV during compression of the working fluid, V_{TV} – initial value of the volume of TV at x_0 at time of formation of the TV.

Parameters of a gear pump used in the experiments are as follows:radius of the base circle – $r_0 = 25{,}84$ mm; circular pitch – $t_0 = 4{,}76$ mm; pressure angle – $\alpha = 27{,}75°$; profile angle – $\alpha_0 = 20°$; number of teeth – $z = 11$; radius of root circle – $R_i = 22{,}7$ mm; addendum radius – $R_e = 34{,}7$ mm; pitch – $m = 5$; angle of the top of involute profile – $\gamma_e = 45{,}59°$;

The data of volume changes of TV as a function of placement point on line of action, calculated by the above method, obtained for the given pump are summarized in Table.

The table shows data obtained by the authors of methods for comparison. Graph of pressure in the TV, depending on the position of the point of gearing in the line of action was constructed using this data. Theoretical calculations show a significant increase of pressure in TV due to the fact that the magnitude of the volume of TV varies widely from its inception to the point at which it reaches minimum value.

Studying the materials of high speed video recording of gear pump working process it was observed that at the increasing of trapped volume cavitation phenomena occurs therein. Since cavitation occurs at a pressure equal to the vapour pressure of the liquid (e.g. water equals 0,03 MPa), it was assumed that in this region the pressure decreases to values of vapour pressure shown on the graph displaying pressure in the trapped volume, depending on the position of contact point on action line (Fig. 6).

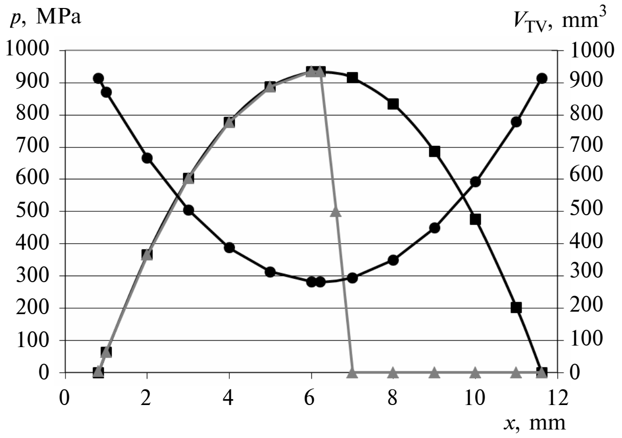
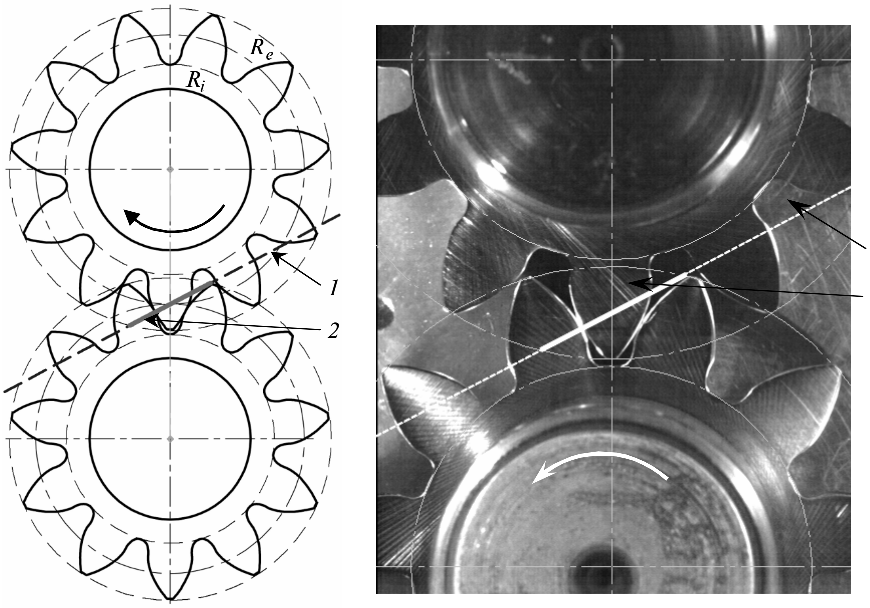


Fig. 6. Graphs of change trapped volume (–●–), pressure in trapped volume (–■–) depending on the position of the contact point of teeth – x along the line of action; –▲– – hypothetical dependence of the pressure in the trapped volume consideration detected cavitation zones

Active part of the line of action is limited by circles of gears addendum. The line passes through the pitch point, angle formed by the line of action and tangent to the pitch circle is equal $27{,}75°$ –

Table. The results of calculations of changes of geometrical parameters of the trapped volume

Geometrical parameters of gearing	$z_1 = z_2 = 12$, $\xi = 0{,}5745$ [5]	$z_1 = z_2 = 11$, $\xi = 0{,}41$
Pitch – m	5	5
Onset of trapped volume formation – x_0, mm	7,046	0,8134
Position, when TV becomes minimal – $x_{\min}$, mm	8,7916	6,2156
Position, when TV disappears – x_k, mm	10,5271	11,6379
Period of TV existence , $x_k - x_0$, mm	3,491	10,8045
Overlap ratio – ε	1,2365	1,2960
Value of the TV area at the moment of formation – $S_{\max}$, mm²	22,0766	24,0689
Minimum value of TV area – $S_{\min}$, mm²	20,4812	7,3988
Value of TV area at the moment of disappearance – $S_{\max}$, mm²	22,0766	24,0689
Difference between values of TV area at the moment of formation and at the moment of disappearance – ΔS, mm²	1,5953	16,6700
Difference between values of TV at the moment of formation and minimal value of TV, if gear width equals b – ΔV, mm³	35,0966 $b = 22$ mm	633,4626 $b = 38$ mm
Pressure excess in TV – Δp, MPa	97,6	935,0



a *b*

Fig. 7. Building line of action for the test pump: *a* — in the drawing; *b* — on video frame: *1* — line of action, *2* — active part of line of action

is the pressure angle α. By combining this sketch with the frame of recorded video, you can define a point on the active part of the line of action in which occurs the formation of the trapped volume, formation and growth of cavities and opening of trapped volume. Fig. 7 shows the location of the line of action for given pump.

In order to bring relations to a dimensionless form set of parameters which presumably depends on the pressure in trapped volume of gear pump was selected. It can be written as follows:

$$p_v = f(\omega, h, \Delta p, l, Q, \nu, \rho, E),$$

where p_v — pressure in trapped volume; ω — rotational speed of gears, s^{-1}; h — tooth height, m; Δp — pressure drop at the inlet and outlet of the pump, Pa; l — length of line of action, m; Q — flowrate through the pump, m^3/s; ν — kinematic viscosity, m^2/s; ρ — fluid density, kg/m^3; E — modulus of elasticity of the liquid, Pa.

Further, π theorem, which states that any equation relating m physical variables, and among them there are n values, which have independent physical units, can be transformed into an equation that link $m-n$ dimensionless complexes composed of these units [7] was used. As the independent physical units ω, h, ρ accordingly rotational speed of gears, tooth height and fluid density were chosen.

As a result, the following complexes were obtained:

$$A_1 = \frac{\Delta p}{\rho h},\ A_2 = \frac{l}{h},\ A_3 = \frac{Q}{\omega h^3},$$

$$A_4 = \frac{\nu}{\omega h^2},\ A_5 = \frac{E}{\rho h},\ A_6 = \frac{p_v}{\rho h}.$$

Thus, the pressure in the vacuum areas in the criterion form is a function of the following criteria

$$\frac{p_v}{\rho h} = f\left(\frac{\Delta p}{\rho h}; \frac{l}{h}; \frac{Q}{\omega h^3}; \frac{\nu}{\omega h^2}; \frac{E}{\rho h}\right)$$

i.e. pressure p_v is a function of the position of point of contact of the teeth in the line of action, centrifugal Reynolds number $\left(\frac{\nu}{\omega h^2}\right)$ and the conditions for entry and exit from this zone $\frac{\Delta p}{\rho h}$.

Fig. 8 shows the criterial dependence between obtained complexes $\frac{p_v}{\rho h}$ and $\frac{l}{h}$.

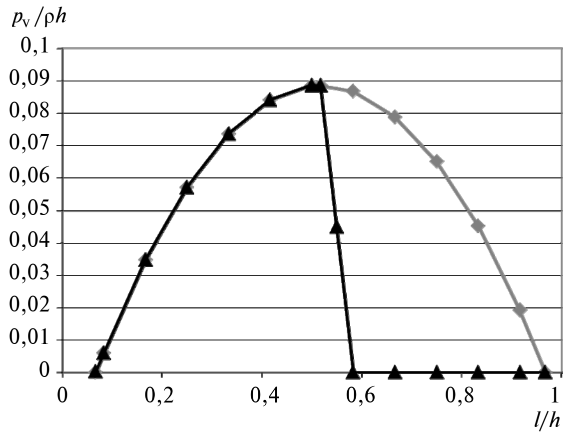


Fig. 8. Criterial dependence $p_v/\rho h$ and l/h: —◆— — theoretical, —▲— — consideration of detected cavitation zones

As can be seen from the figure, this dependence has its extreme value at a point corresponding $\frac{l}{h} = 0,6$.

Conclusions

Investigations aimed at obtaining flow pattern in the involute external gear pump showed that in trapped volume, which formed in the operation of a gear pump, occurs cavitation phenomena were conducted. Their rate depends on factors such as the pressure in the suction chamber and the gears rotational speed. Examination of the dynamics of bubble growth showed that its nucleation occurs at increasing of trapped volume. Comparison of the placement of the gear teeth contact point on the video frame with the results obtained by the reviewed methods confirmed the correctness of the calculated data. The dependence, which expresses the pressure in the trapped volume in dimensionless form was obtained.

Further investigations suggest study of the characteristics of fluid flow behind gear pump and the influence on it cavitation phenomena described in this paper.

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