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WIND TURBINE POWER CURVE EXPONENTIAL MODEL WITH DIFFERENTIABLE CUT-IN AND CUT-OUT PARTS

Background. The main characteristic of a wind turbine is its power curve. Getting measurement data off powerful wind turbines is a way harder than measuring characteristics of wind turbines for individual/home use. A yet significant gap is that all wind turbines have a few similarities in their power curves but they do not have a formalized description, which could help in selecting better turbines fitting specific areas (without precise measurements in a vicinity of cut-in and cut-out speeds).
Objective. As there is a straight lack of mathematical description of wind turbine power curves, the goal is to obtain a model of such curves.
Methods. A power curve is of seven parts. Factual power curves remotely remind trapezia with curvilinear flanks. Because of inertia, the curvilinearity is severer for those wind turbines whose output power is greater. As blades of industrial wind turbines are too massive, their inertia makes those lag effects, that could be modeled by using natural smoothness of power curves. For describing that smoothness along with the curvilinearity, we use two increasing and two decreasing exponential functions for the flanks.
Results. A wind turbine power output function consists of two zero parts, one rated-out part, and the suggested four exponential parts. The cut-in parts are described with two increasing exponential functions whose exponential growth factors are equal. The cut-out parts are described with two decreasing exponential functions whose exponential decrease factors are equal also. Such equal factors ensure strong differentiability of the power curve within those parts.
Conclusions. The exponential model is for a general description of the wind turbine power curve. Having differentiable cut-in and cut-out parts, it suggests the "natural smoothing" that happens in reality due to highly-inertial wind turbine blades. The model is not necessarily to be used to fit some experimental data, but rather for patterning power curves.
Keywords: wind turbine; power curve; cut-in speed; cut-out speed; natural smoothness; exponential curve.

Introduction

Wind power is the most promising source of renewable energy. It is produced by wind farms consisting of wind turbines [1, 2]. The wind farm productivity is mainly determined by characteristics of those turbines imposed on wind statistics of an area where the farm is built [1, 3, 4]. While the wind statistics cannot be influenced and changed, the wind turbine with better characteristics can be selected for fitting the area wind statistics [5, 6]. Thus, before deploying a wind farm, the wind turbine characteristics should be modeled [4, 5, 7, 8].

The main characteristic of a wind turbine is its power curve. This is a wind turbine power output function $w(s)$ that shows an amount of kilowatts/megawatts produced at a range of wind speeds s. This range might be called active. When a power curve is plotted, the wind speed changes slowly, not abruptly. Acceleration of the wind speed change (caused by wind gusts, blasts, hurricanes, etc.) badly influences on the power output, but it has been studied less [9, 10]. As close to an ideal case, the power curve would be rectangular or trapezoid having very steep flanks. The left flank, where the output power starts off zero, appears at a cut-in wind speed. The right flank, where the output power drops down back to zero, appears at a cut-out wind speed [1, 6]. Factual power curves remotely remind trapezia with curvilinear flanks, whereon the nominal (rated-out) power appears at a speed greater than a cut-in speed, and the real drop to zero (a wind turbine turn-off) appears at a speed greater than a cut-out speed.

A few examples of power curves in [6] are of industrial turbines. For those wind turbines whose output power is greater, the left flank becomes more gently sloping. Their power curves remind trapezia less. And the right flanks become not so steep at all. An explanation is the turbines of the greater power are bigger, and their huger blades are much massive and inertial, taking longer periods to slow down and stop. Obviously, getting measurement data off more powerful wind turbines is a way harder than measuring characteristics of wind turbines for individual/home use. All the more that

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wind farms are not built of small turbines, whereas individual/home (non-industrial) use does not require building wind farms.

A yet significant gap is that all wind turbines have a few similarities in their power curves but they do not have a formalized description, which could help in selecting better turbines fitting specific areas (without precise measurements in a vicinity of cut-in and cut-out speeds). Indeed, it would be far easier to get a power curve knowing just a rated-out power along with cut-in and cut-out wind speeds. Then a whole wind farm is possible to be modeled for its power output depending on wind statistics of a given area [5, 6].

However, one should be aware of the wind power curve depends on the wind speed acceleration and turbulence [4, 10, 11]. The power curve is locally distorted worse as the wind speed acceleration grows higher. Apparently, small accelerations and turbulence always exist, whichever area for the wind farm is chosen. But they are very specific for different areas. This is why the known approaches to modeling power curves being tied to data of a given area are not correct for a general use. For instance, the power curves in [6] represented officially in product descriptions will pretty change for every turbine type on different areas. The change will be more considerable for Vestas V112-3.0 MW [2, 3, 10].

There are parametric and non-parametric techniques in power curve modeling methodology [11, 12]. According to [11, 13], the models based on the concept of power available in the wind, like the probabilistic and polynomial models, do not give accurate results. This is because of the fact that the fraction of wind power that is converted to electrical power depends on several other parameters like rotational speed of the turbine, turbine blade parameters, and the efficiencies of the mechanical transmission system and generator efficiency. The models based exclusively on the shape of the power curve, like the linearized segmented model and the model with the Weibull's parameters, perform poorly because the performance of the wind turbines with different design parameters and ratings cannot be modeled using a single set of general equations [11].

The modeling methods in which characteristic equations are developed based on the actual power curve of the wind turbine appear the best for wind resource estimation and for identifying potential wind farm sites/areas [4, 5, 11, 12]. This also aids to make the right choice of turbines while the wind farm is projected [6]. Nevertheless, the actual power curve requires long antecedent data on the wind speed and turbine power. Thus, a power prediction without the historic data should be based on some generalizations, without tying to a definite site/area. If a sizeable number of training and testing data is available, then non-parametric techniques based on data mining techniques and neural networks perform well. The performance of the wind turbine power curve modeled using four and five parameter logistic expressions is reported to outperform the linearized segmented model and the models based on neural network, fuzzy logic and data mining algorithms [9, 11, 13, 14]. Thus, it is expected that such a outperformance shall exist for poorer initial and antecedent data.

Problem statement

As there is a straight lack of mathematical description of wind turbine power curves in the case of missing long antecedent data, the goal is to obtain a model of such curves. Such a model shall allow plotting a power curve without direct measurements but only by giving a nominal power along with a few crucial points (e. g., amounts of kilowatts/megawatts produced at cut-in and cut-out wind speeds). For achieving the said goal, the three tasks are to be accomplished:

1. To list those crucial points and initial data that should be given before modeling.

2. Based on the list, to substantiate a consistent generalized model of the wind turbine power curve compatible with power curves of real turbines.

3. To show how well the substantiated model fits some real measurements.

The list of initial data is presumed to be a set of input arguments for making the model. The said compatibility with real turbines' power curves implies not only true fits, but smoothing the right flank also, because that steepness must disappear after zooming in. Within the item #3, a procedure of how to fit the measurements will be explained. After that, the suggested model shall be discussed and a conclusion will be given with an outlook for further research.

A set of input arguments for the wind turbine power curve model

A power curve is of seven parts (see them in the three examples for industrial turbines in Fig. 1 and in Fig. 2). Emphasizing the two zero-power lines and the nominal power line is apparent. Both

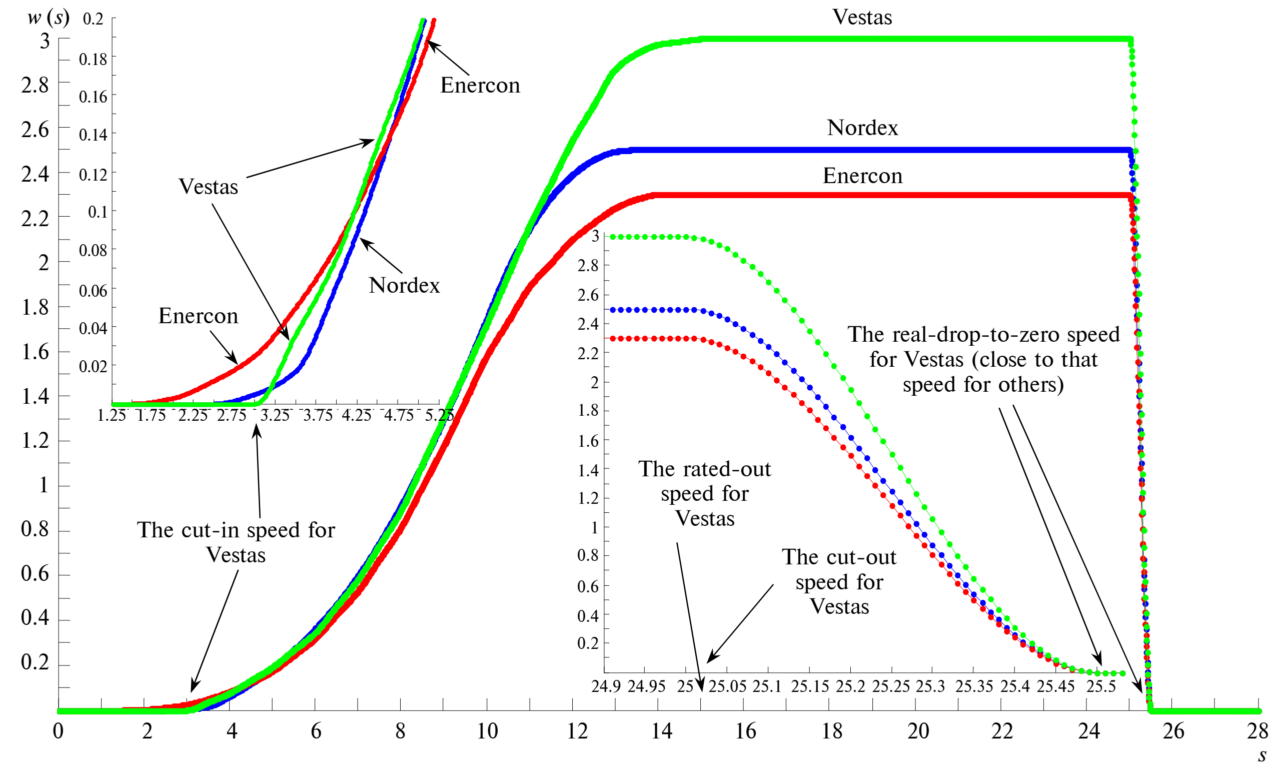

Fig. 1. Power curves (represented officially in product descriptions) of wind turbines Enercon E82 E2 (2.3 MW), Nordex N90/2500 (2.5 MW), Vestas V112-3.0 MW; bold points are those at which real measurements were conducted

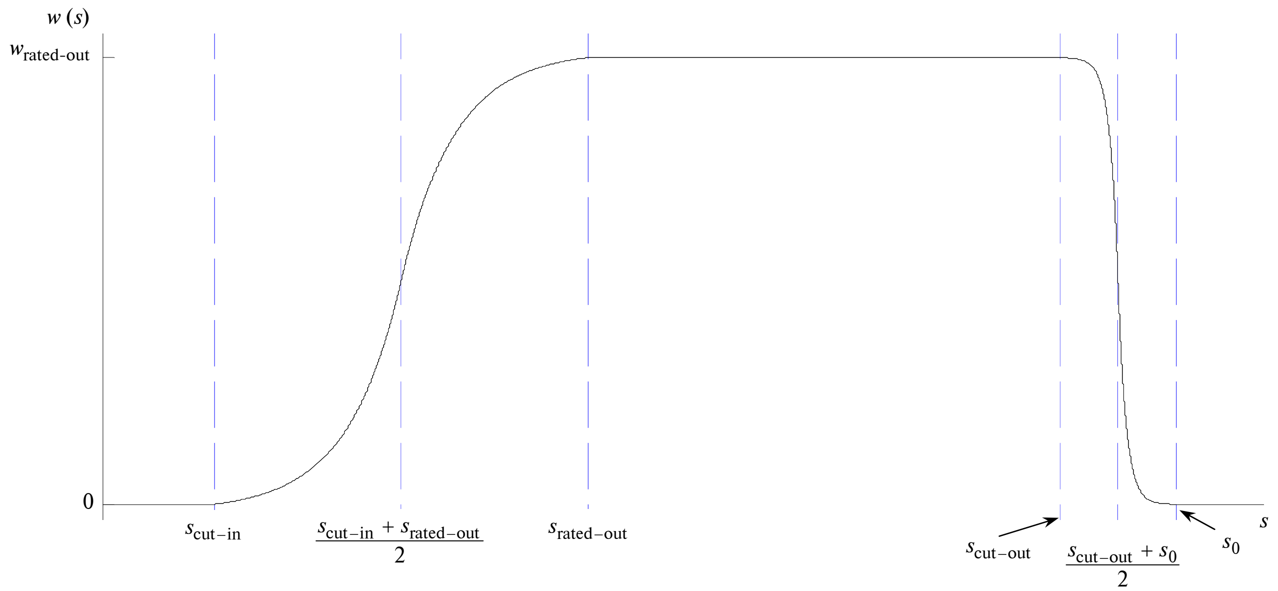

Fig. 2. A sketch of the wind turbine power curve with its emphasized parts

the flanks are divided into two parts. Ascending off the zero and ascending up to the nominal power are on the left flank (the cut-in parts). Descending off the nominal power and descending down to the zero are on the right flank (the cut-out parts).

It is well seen from Fig. 2 that the crucial points are: cut-in speed $s_{\text{cut-in}}$, rated-out speed $s_{\text{rated-out}}$, cut-out speed $s_{\text{cut-out}}$, and real-drop-to-zero speed s_0. The second and third parts can be separated by a simple middle point

$$s = \frac{s_{\text{cut-in}} + s_{\text{rated-out}}}{2}.$$

Similarly, the fifth and sixth parts can be separated by wind speed

$$s = \frac{s_{\text{cut-out}} + s_0}{2}.$$

This is very naive, but the most expedient.

While those crucial wind speeds may be unofficially changed for fitting the model best, the nominal (rated-out) power $w_{\text{rated-out}}$ is given fixed. It is of the initial data wherein steepnesses for both flanks may be given as well.

An exponential model of the wind turbine power curve

The wind turbine power at each wind speed is influenced by a great deal of factors, both stochastic and predetermined [2, 3, 7, 11, 13]. The curvilinearity of the power curve flanks is implicitly formed in a similar way. That can be thought of as it is an average of a stochastic process with a normal distribution. All the more that logistic functions suit very accurately for the curvilinearity [4, 9, 11, 12, 14]. That is why the curvilinearity is going to be generally described with exponents.

Based on the list of crucial points and the sketch in Figure 2, a wind turbine power output function is stated implicitly as

$$w(s) = \begin{cases} 0, \; s \leq s_{\text{cut-in}}, \\ w_1(s), \; s \in \left[s_{\text{cut-in}}; \; \frac{s_{\text{cut-in}} + s_{\text{rated-out}}}{2} \right], \\ w_2(s), \; s \in \left[\frac{s_{\text{cut-in}} + s_{\text{rated-out}}}{2}; s_{\text{rated-out}} \right], \\ w_{\text{rated-out}}, \; s \in \left[s_{\text{rated-out}}; \; s_{\text{cut-out}} \right], \\ w_3(s), \; s \in \left[s_{\text{cut-out}}; \; \frac{s_{\text{cut-out}} + s_0}{2} \right], \\ w_4(s), \; s \in \left[\frac{s_{\text{cut-out}} + s_0}{2}; \; s_0 \right], \\ 0, \; s \geq s_0, \end{cases} \tag{1}$$

having six joint points

$$s_{\text{cut-in}}, \; \frac{s_{\text{cut-in}} + s_{\text{rated-out}}}{2}, \; s_{\text{rated-out}}, \; s_{\text{cut-out}}, \; \frac{s_{\text{cut-out}} + s_0}{2}, \; s_0 .$$

On the second and third parts of function (1) we have:

$$w_1(s_{\text{cut-in}}) = 0, \tag{2}$$

$$w_1\left(\frac{s_{\text{cut-in}} + s_{\text{rated-out}}}{2}\right) = \frac{w_{\text{rated-out}}}{2}, \tag{3}$$

$$w_2\left(\frac{s_{\text{cut-in}} + s_{\text{rated-out}}}{2}\right) = \frac{w_{\text{rated-out}}}{2}, \tag{4}$$

$$w_2(s_{\text{rated-out}}) = w_{\text{rated-out}}. \tag{5}$$

As blades of industrial wind turbines are too massive, their inertia makes those lag effects, that could be modeled by using natural smoothness of power curves. For describing that smoothness, we can obviously take two increasing exponential functions:

$$w_1(s) = (\exp((s - s_{\text{cut-in}}) r_{\text{cut-in}}) - 1) K_1 \tag{6}$$

by some exponential growth factor $r_{\text{cut-in}}$ and

$$w_2(s) = \frac{w_{\text{rated-out}}}{2} + \left(1 - \exp\left(\left(\frac{s_{\text{cut-in}} + s_{\text{rated-out}}}{2} - s\right) r_{\text{rated-out}}\right)\right) K_2 \tag{7}$$

by some exponential growth factor $r_{\text{rated-out}}$. Clearly, function (6) satisfies condition (2), and function (7) satisfies condition (4). Constants K_1 and K_2 are found from conditions (3) and (5). Plugging (3) into (6) gives us:

$$w_1\left(\frac{s_{\text{cut-in}} + s_{\text{rated-out}}}{2}\right) = \left(\exp\left(\left(\frac{s_{\text{cut-in}} + s_{\text{rated-out}}}{2} - s_{\text{cut-in}}\right) \times r_{\text{cut-in}}\right) - 1\right) K_1 = \frac{w_{\text{rated-out}}}{2},$$

whence

$$K_1 = \frac{w_{\text{rated-out}}}{2 \cdot \left(\exp\left(\frac{s_{\text{rated-out}} - s_{\text{cut-in}}}{2} \cdot r_{\text{cut-in}}\right) - 1\right)}$$

and, subsequently,

$$w_1(s) = \frac{w_{\text{rated-out}}}{2 \cdot \left(\exp\left(\frac{s_{\text{rated-out}} - s_{\text{cut-in}}}{2} \cdot r_{\text{cut-in}}\right) - 1\right)}$$

$$\times (\exp((s - s_{\text{cut-in}}) r_{\text{cut-in}}) - 1). \qquad (8)$$

Plugging (5) into (7) gives us:

$$w_2(s_{\text{rated-out}}) = \frac{w_{\text{rated-out}}}{2} + \left(1 - \exp\left(\left(\frac{s_{\text{cut-in}} + s_{\text{rated-out}}}{2} - s_{\text{rated-out}}\right) \times r_{\text{rated-out}}\right)\right) K_2 = w_{\text{rated-out}},$$

whence

$$K_2 = \frac{w_{\text{rated-out}}}{2 \cdot \left(1 - \exp\left(\frac{s_{\text{cut-in}} - s_{\text{rated-out}}}{2} \cdot r_{\text{rated-out}}\right)\right)}$$

and, subsequently,

$$w_2(s) = \frac{w_{\text{rated-out}}}{2} + \frac{w_{\text{rated-out}}}{2 \cdot \left(1 - \exp\left(\frac{s_{\text{cut-in}} - s_{\text{rated-out}}}{2} \cdot r_{\text{rated-out}}\right)\right)} \times \left(1 - \exp\left(\left(\frac{s_{\text{cut-in}} + s_{\text{rated-out}}}{2} - s\right) r_{\text{rated-out}}\right)\right). \qquad (9)$$

On the fifth and sixth parts of function (1) we have:

$$w_3(s_{\text{cut-out}}) = w_{\text{rated-out}}, \qquad (10)$$

$$w_3\left(\frac{s_{\text{cut-out}} + s_0}{2}\right) = \frac{w_{\text{rated-out}}}{2}, \qquad (11)$$

$$w_4\left(\frac{s_{\text{cut-out}} + s_0}{2}\right) = \frac{w_{\text{rated-out}}}{2}, \qquad (12)$$

$$w_4(s_0) = 0. \qquad (13)$$

For these parts we can take two decreasing exponential functions similar to (6) and (7):

$$w_3(s) = w_{\text{rated-out}} + (1 - \exp((s - s_{\text{cut-out}}) r_{\text{cut-out}})) K_3 \qquad (14)$$

by some exponential decrease factor $r_{\text{cut-out}}$ and

$$w_4(s) = \frac{w_{\text{rated-out}}}{2} + \left(\exp\left(\left(\frac{s_{\text{cut-out}} + s_0}{2} - s\right) r_0\right) - 1\right) K_4 \qquad (15)$$

by some exponential decrease factor r_0. Clearly, function (14) satisfies condition (10), and function (15) satisfies condition (12). Constants K_3 and K_4 are found from conditions (11) and (13). Plugging (11) into (14) gives us:

$$w_3\left(\frac{s_{\text{cut-out}} + s_0}{2}\right) = w_{\text{rated-out}} + \left(1 - \exp\left(\left(\frac{s_{\text{cut-out}} + s_0}{2} - s_{\text{cut-out}}\right) \times r_{\text{cut-out}}\right)\right) K_3 = \frac{w_{\text{rated-out}}}{2},$$

whence

$$K_3 = \frac{w_{\text{rated-out}}}{2 \cdot \left(\exp\left(\frac{s_0 - s_{\text{cut-out}}}{2} \cdot r_{\text{cut-out}}\right) - 1\right)}$$

and, subsequently,

$$w_3(s) = w_{\text{rated-out}} + \frac{w_{\text{rated-out}}}{2 \cdot \left(\exp\left(\frac{s_0 - s_{\text{cut-out}}}{2} \cdot r_{\text{cut-out}}\right) - 1\right)} \times (1 - \exp((s - s_{\text{cut-out}}) r_{\text{cut-out}})). \qquad (16)$$

And, finally, plugging (13) into (15) gives us:

$$w_4(s_0) = \frac{w_{\text{rated-out}}}{2} + \left(\exp\left(\left(\frac{s_{\text{cut-out}} + s_0}{2} - s_0\right) r_0\right) - 1\right) K_4 = 0,$$

whence

$$K_4 = \frac{w_{\text{rated-out}}}{2 \cdot \left(1 - \exp\left(\frac{s_{\text{cut-out}} - s_0}{2} \cdot r_0\right)\right)}$$

and, subsequently,

$$w_4(s) = \frac{w_{\text{rated-out}}}{2} + \frac{w_{\text{rated-out}}}{2 \cdot \left(1 - \exp\left(\frac{s_{\text{cut-out}} - s_0}{2} \cdot r_0\right)\right)}$$

$$\times\left(\exp\left(\left(\frac{s_{\text{cut-out}}+s_0}{2}-s\right)r_0\right)-1\right). \qquad (17)$$

Factors $r_{\text{cut-in}}$, $r_{\text{rated-out}}$, $r_{\text{cut-out}}$, r_0 may be theoretically given as the initial data. However, their magnitudes are going to be constrained to a property of strong differentiability.

Strong differentiability for the flanks' proper curvilinearity

For completing the "natural smoothing", second and third parts of function (1), and fifth and sixth parts of this function should be differentiable within intervals, on which they are defined (except, probably, for endpoints of those intervals). Therefore, equalities

$$\left.\frac{dw_1(s)}{ds}\right|_{s=\frac{s_{\text{cut-in}}+s_{\text{rated-out}}}{2}}=\left.\frac{dw_2(s)}{ds}\right|_{s=\frac{s_{\text{cut-in}}+s_{\text{rated-out}}}{2}} \qquad (18)$$

and

$$\left.\frac{dw_3(s)}{ds}\right|_{s=\frac{s_{\text{cut-out}}+s_0}{2}}=\left.\frac{dw_4(s)}{ds}\right|_{s=\frac{s_{\text{cut-out}}+s_0}{2}} \qquad (19)$$

should hold. Equalities (18) and (19) are principally important for making the power curve flanks smoothly/properly curvilinear.

For equality (18), the derivative of function (8) is

$$\frac{dw_1(s)}{ds}$$
$$=\frac{w_{\text{rated-out}}\cdot r_{\text{cut-in}}}{2\cdot\left(\exp\left(\frac{s_{\text{rated-out}}-s_{\text{cut-in}}}{2}\cdot r_{\text{cut-in}}\right)-1\right)}$$
$$\times\exp((s-s_{\text{cut-in}})r_{\text{cut-in}}),$$

so

$$\left.\frac{dw_1(s)}{ds}\right|_{s=\frac{s_{\text{cut-in}}+s_{\text{rated-out}}}{2}}$$
$$=\frac{w_{\text{rated-out}}\cdot r_{\text{cut-in}}}{2\cdot\left(\exp\left(\frac{s_{\text{rated-out}}-s_{\text{cut-in}}}{2}\cdot r_{\text{cut-in}}\right)-1\right)}$$
$$\times\exp\left(\frac{s_{\text{rated-out}}-s_{\text{cut-in}}}{2}\cdot r_{\text{cut-in}}\right). \qquad (20)$$

The derivative of function (9) is

$$\frac{dw_2(s)}{ds}$$
$$=\frac{w_{\text{rated-out}}\cdot r_{\text{rated-out}}}{2\cdot\left(1-\exp\left(\frac{s_{\text{cut-in}}-s_{\text{rated-out}}}{2}\cdot r_{\text{rated-out}}\right)\right)}$$
$$\times\exp\left(\left(\frac{s_{\text{cut-in}}+s_{\text{rated-out}}}{2}-s\right)r_{\text{rated-out}}\right),$$

so

$$\left.\frac{dw_2(s)}{ds}\right|_{s=\frac{s_{\text{cut-in}}+s_{\text{rated-out}}}{2}}$$
$$=\frac{w_{\text{rated-out}}\cdot r_{\text{rated-out}}}{2\cdot\left(1-\exp\left(\frac{s_{\text{cut-in}}-s_{\text{rated-out}}}{2}\cdot r_{\text{rated-out}}\right)\right)}. \qquad (21)$$

It is easy to see that values (20) and (21) can be equal only when $r_{\text{cut-in}}=r_{\text{rated-out}}$. Then, indeed,

$$\left.\frac{dw_2(s)}{ds}\right|_{s=\frac{s_{\text{cut-in}}+s_{\text{rated-out}}}{2}}$$
$$=\frac{w_{\text{rated-out}}\cdot r_{\text{cut-in}}}{2\cdot\left(1-\exp\left(\frac{s_{\text{cut-in}}-s_{\text{rated-out}}}{2}\cdot r_{\text{cut-in}}\right)\right)}$$
$$\times\frac{\exp\left(\frac{s_{\text{rated-out}}-s_{\text{cut-in}}}{2}\cdot r_{\text{cut-in}}\right)}{\exp\left(\frac{s_{\text{rated-out}}-s_{\text{cut-in}}}{2}\cdot r_{\text{cut-in}}\right)}$$
$$=\left.\frac{dw_1(s)}{ds}\right|_{s=\frac{s_{\text{cut-in}}+s_{\text{rated-out}}}{2}}.$$

For equality (19), the derivative of function (16) is

$$\frac{dw_3(s)}{ds}=\frac{w_{\text{rated-out}}\cdot r_{\text{cut-out}}}{2\cdot\left(1-\exp\left(\frac{s_0-s_{\text{cut-out}}}{2}\cdot r_{\text{cut-out}}\right)\right)}$$
$$\times\exp((s-s_{\text{cut-out}})r_{\text{cut-out}}),$$

so

$$\left.\frac{dw_3(s)}{ds}\right|_{s=\frac{s_{\text{cut-out}}+s_0}{2}}$$
$$=\frac{w_{\text{rated-out}}\cdot r_{\text{cut-out}}}{2\cdot\left(1-\exp\left(\frac{s_0-s_{\text{cut-out}}}{2}\cdot r_{\text{cut-out}}\right)\right)}\times$$

$$\times \exp\left(\frac{s_0 - s_{\text{cut-out}}}{2} \cdot r_{\text{cut-out}}\right). \tag{22}$$

The derivative of function (17) is

$$\frac{dw_4(s)}{ds} = \frac{w_{\text{rated-out}} \cdot r_0}{2 \cdot \left(\exp\left(\frac{s_{\text{cut-out}} - s_0}{2} \cdot r_0\right) - 1\right)}$$

$$\times \exp\left(\left(\frac{s_{\text{cut-out}} + s_0}{2} - s\right) r_0\right),$$

so

$$\left.\frac{dw_4(s)}{ds}\right|_{s=\frac{s_{\text{cut-out}}+s_0}{2}}$$

$$= \frac{w_{\text{rated-out}} \cdot r_0}{2 \cdot \left(\exp\left(\frac{s_{\text{cut-out}} - s_0}{2} \cdot r_0\right) - 1\right)}. \tag{23}$$

Obviously, values (20) and (21) can be equal only when $r_{\text{cut-out}} = r_0$. Then, indeed,

$$\left.\frac{dw_4(s)}{ds}\right|_{s=\frac{s_{\text{cut-out}}+s_0}{2}}$$

$$= \frac{w_{\text{rated-out}} \cdot r_0}{2 \cdot \left(\exp\left(\frac{s_{\text{cut-out}} - s_0}{2} \cdot r_0\right) - 1\right)}$$

$$\times \frac{\exp\left(\frac{s_0 - s_{\text{cut-out}}}{2} \cdot r_0\right)}{\exp\left(\frac{s_0 - s_{\text{cut-out}}}{2} \cdot r_0\right)} = \left.\frac{dw_3(s)}{ds}\right|_{s=\frac{s_{\text{cut-out}}+s_0}{2}}.$$

Note that function (1), even with parts (8), (9), (16), (17), by $r_{\text{cut-in}} = r_{\text{rated-out}}$ and $r_{\text{cut-out}} = r_0$, is still not differentiable at points

$$s_{\text{cut-in}},\ s_{\text{rated-out}},\ s_{\text{cut-out}},\ s_0. \tag{24}$$

Nevertheless, leaps of the first derivative at each of those points will be insignificant by appropriate values $r_{\text{cut-in}}$ and r_0. All the more that differentiability at points (24) has a very little impact on the curvilinearity of the power curve flanks.

Fitting real measurements

If we have N measurements $\{\tilde{w}(s_k)\}_{k=1}^{N}$ of a wind turbine power output, then let $\tilde{w}(s)$ be a polyline linking all those points. A theoretical power curve $w(s)$ having a set of arguments

$$A = \{s_{\text{cut-in}}, s_{\text{rated-out}}, r_{\text{cut-in}}, s_{\text{cut-out}}, s_0, r_0\} \tag{25}$$

is adjusted to those measurements. In other words, a curve

$$w^*(s) \in \arg\left(\min_{\substack{w(s)\in\bar{\mathbb{W}}_2[s_{\text{cut-in}};\, s_0] \\ \subset\mathbb{L}_2[s_{\text{cut-in}};\, s_0]}} \|w(s) - \tilde{w}(s)\|\right)$$

$$= \arg\left(\min_{w(s)\in\bar{\mathbb{W}}_2[s_{\text{cut-in}};\, s_0]\subset\mathbb{L}_2[s_{\text{cut-in}};\, s_0]}\right.$$

$$\left.\left(\sqrt{\int_{s_{\text{cut-in}}}^{s_0} (w(s) - \tilde{w}(s))^2 ds}\right)\right) \tag{26}$$

is found by a subspace

$$\bar{\mathbb{W}}_2[s_{\text{cut-in}};\, s_0] \subset \mathbb{L}_2[s_{\text{cut-in}};\, s_0]$$

consisting of functions (1) with parts (8), (9), (16), (17), by $r_{\text{cut-in}} = r_{\text{rated-out}}$ and $r_{\text{cut-out}} = r_0$, where set (25) is applied under the minimum in (26). Function $w^*(s)$ in (26) is the best approximation to $\tilde{w}(s)$. It can be found easier in two stages. Owing to that the power output is constant on the interval $[s_{\text{rated-out}};\, s_{\text{cut-out}}]$, parts

$$w_{\text{cut-in}}^{\langle A_{\text{in}}\rangle}(s) \in \arg$$

$$\left(\min_{w_{\text{cut-in}}(s)\in\bar{\mathbb{W}}_2[s_{\text{cut-in}};\, s_{\text{rated-out}}]\subset\mathbb{L}_2[s_{\text{cut-in}};\, s_{\text{rated-out}}]}\right.$$

$$\left.\left(\sqrt{\int_{s_{\text{cut-in}}}^{s_{\text{rated-out}}} \left(w_{\text{cut-in}}(s) - \tilde{w}(s)\right)^2 ds}\right)\right) \tag{27}$$

and

$$w_{\text{cut-out}}^{\langle A_{\text{out}}\rangle}(s) \in \arg\left(\min_{w_{\text{cut-out}}(s)\in\bar{\mathbb{W}}_2[s_{\text{cut-out}};\, s_0]\subset\mathbb{L}_2[s_{\text{cut-out}};\, s_0]}\right.$$

$$\left.\left(\sqrt{\int_{s_{\text{cut-out}}}^{s_0} \left(w_{\text{cut-out}}(s) - \tilde{w}(s)\right)^2 ds}\right)\right) \tag{28}$$

are found separately/independently applying the respective subsets

$$A_{\text{in}} = \{s_{\text{cut-in}}, s_{\text{rated-out}}, r_{\text{cut-in}}\} \tag{29}$$

and

$$A_{\text{out}} = \{s_{\text{cut-out}}, s_0, r_0\} \tag{30}$$

of set (25). Then

$$w^{**}(s) = \begin{cases} 0, \ s \le s_{\text{cut-in}} \\ w_{\text{cut-in}}^{\langle A_{\text{in}} \rangle}(s), \ s \in [s_{\text{cut-in}}; \ s_{\text{rated-out}}] \\ w_{\text{rated-out}}, \ s \in [s_{\text{rated-out}}; \ s_{\text{cut-out}}] \\ w_{\text{cut-out}}^{\langle A_{\text{out}} \rangle}(s), \ s \in [s_{\text{cut-out}}; \ s_0] \\ 0, \ s \ge s_0 \end{cases} \quad (31)$$

belonging to the functional subspace

$$\bar{\mathbb{W}}_2[0; \infty) \subset \mathbb{L}_2[0; \infty).$$

Clearly, $w^{**}(s) = w^{*}(s)$ if to consider function (31) on just the subspace

$$\bar{\mathbb{W}}_2[s_{\text{cut-in}}; s_0] \subset \mathbb{L}_2[s_{\text{cut-in}}; s_0] \subset \mathbb{L}_2[0; \infty).$$

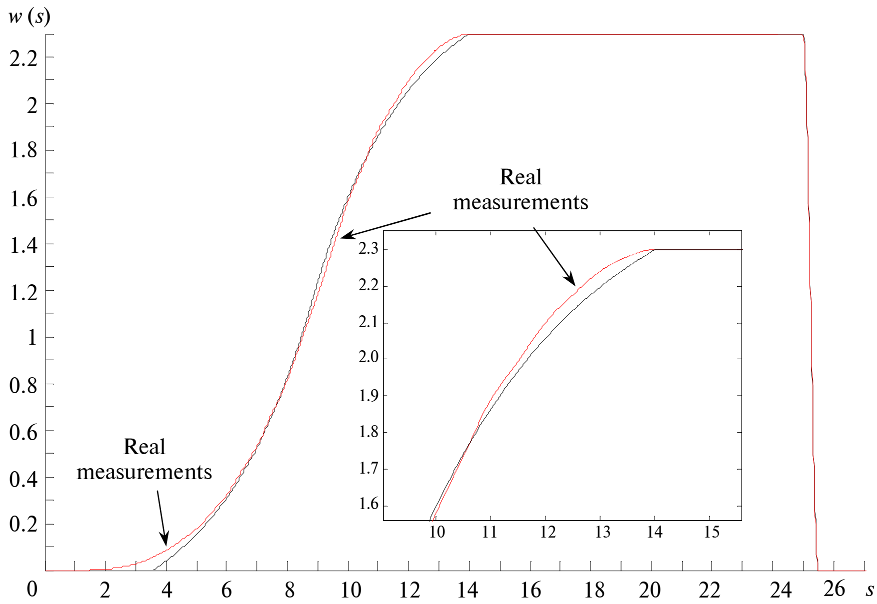

Fig. 3. Fitting a measured power curve of the wind turbine Enercon E82 E2 by $s_{\text{cut-in}} = 3.6$, $s_{\text{rated-out}} = 14.02$, $r_{\text{cut-in}} = 0.32$, $s_{\text{cut-out}} = 25.03$, $s_0 = 25.47$, $r_0 = 4.16$

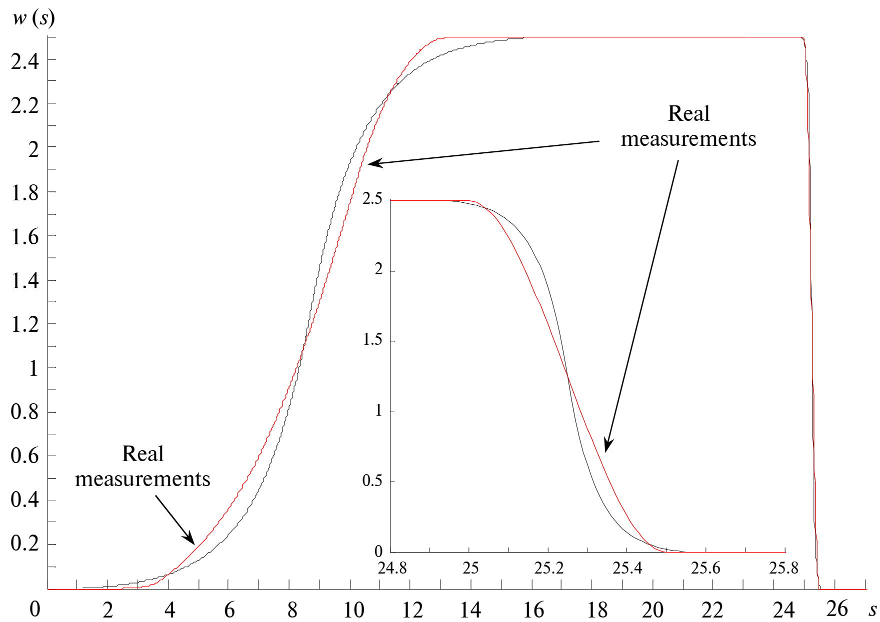

Fig. 4. Fitting a measured power curve of the wind turbine Nordex N90/2500 by $s_{\text{cut-in}} = 1.21$, $s_{\text{rated-out}} = 16.2$, $r_{\text{cut-in}} = 0.59$, $s_{\text{cut-out}} = 24.95$, $s_0 = 25.55$, $r_0 = 13.8$

Figs. 3 and 4 show how exponential model (1) with parts (8), (9), (16), (17), by $r_{\text{cut-in}} = r_{\text{rated-out}}$ and $r_{\text{cut-out}} = r_0$, fits the measured power curves of wind turbines Enercon E82 E2 and Nordex N90/2500, where parameters (25) are easily found by a looped search by a step 0.01 (such accuracy is sufficient). However, fitting at cut-in and rated-out parts does not seem so good everywhere (the right flank in Figure 3) because of sharp changing of the power around the "corners". Consequently, finding a curve by (26) might have been accompanied with an additional condition of that leaps of the first derivative at each of points (24) be minimized.

For fitting more around the "corners", exponential growth/decrease factors may be taken different. Then, obviously, speeds $\frac{s_{\text{cut-in}} + s_{\text{rated-out}}}{2}$ and $\frac{s_{\text{cut-out}} + s_0}{2}$ will not be the joint points. Nevertheless, the generalized model by those equal factors is still consistent because it should rather be "slower" around the "corners" due to probable wind dynamics and turbulence (the plotted power curves are static because their real measurements were conducted while the wind speed was changed in non-dynamic mode).

It is worth to notice that this is not about to fit real measurements at all costs. While parameters (25) are searched/evaluated with their corresponding loops, where subsets (29) and (30) are evaluated separately, a few versions for them may come out. Then a parameter's evaluation that enables rounding the "corners" is accepted. Thus, evaluations of speeds $s_{\text{cut-in}}$ and $s_{\text{cut-out}}$ will be less than their nominal magnitudes (if they are available), and evaluations of speeds $s_{\text{rated-out}}$ and s_0 will be greater than their nominal magnitudes (if they are available). According to the said, the approximation in Figure 4 is more universal, unlike the approximation in Figure 3, which will work accurately on areas with no wind gusts (and where the wind speed changes very slowly).

Discussion

The exponential model reflects the sketch in Figure 2, where the flanks are really overextended. Such an overextension will exist due to the inertia of the blades. Their inertia dramatically grows when wind changes dynamically. Therefore, a static representation of the power curve is substituted with the proposed model suggesting that the strong differentiability is preferred to independently measured states. Despite the non-differentiability at points (24), the power curve exponential model becomes very smooth if appropriate parameters (25) are adjusted.

While adjusting, speeds $s_{\text{cut-in}}$ and $s_{\text{cut-out}}$ may be decreased. Speeds $s_{\text{rated-out}}$ and s_0 may be increased. This leads to that these speeds will differ from those ones declared by a wind turbine manufacturer. However, it is not a demerit but just a modifier.

An essential merit of the proposed model is that its power curve's natural smoothness is totally consistent with the inertia. It concerns especially industrial wind turbines. Eventually, the model allows calculating more reliable amounts of the expected power output based on wind statistics.

Conclusions

Exponential model (1) with parts (8), (9), (16), (17), by $r_{\text{cut-in}} = r_{\text{rated-out}}$ and $r_{\text{cut-out}} = r_0$, is for a general description of the wind turbine power curve. It has differentiable cut-in and cut-out parts. Unlike static power curves plotted mostly by changing the wind speed non-dynamically, the model suggests the "natural smoothing" that happens in reality due to highly-inertial blades (compare the "natural smoothing" in Figure 2 to real-but-static measurements in Figure 1).

If a researcher/projector of a wind farm wants to select the best wind turbines under the known wind statistics, then parameters (25) are varied along with the nominal power until the expected power of the wind farm reaches its maximum (e. g., see [2, 4, 5, 7] by [6]). The corresponding configuration of parameters (25) and $w_{\text{rated-out}}$ is compared to the existing wind turbines, whereupon a closest match is selected.

The research may be furthered with appending the condition of that leaps of the first derivative at each of points (24) be minimized. For this, conventions of $r_{\text{cut-in}} = r_{\text{rated-out}}$ and $r_{\text{cut-out}} = r_0$ will be canceled. One should remember, however, that the model is not necessarily to be used to fit some experimental data, but rather for patterning power curves. Practically, this will do also when a nominal power along with amounts of kilowatts/megawatts produced at cut-in and cut-out wind speeds are given.

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В.В. Романюк

ЭКСПОНЕНЦИАЛЬНАЯ МОДЕЛЬ КРИВОЙ МОЩНОСТИ ВЕТРОВОЙ ТУРБИНЫ С ДИФФЕРЕНЦИРУЕМЫМИ ЧАСТЯМИ НА РЕЖИМАХ ВКЛЮЧЕНИЯ И ВЫКЛЮЧЕНИЯ

Проблематика. Основной характеристикой ветровой турбины является кривая мощности. Сбор данных измерений с мощных ветровых турбин является намного более сложным, чем измерение характеристик ветровых турбин для индивидуального/домашнего использования. Еще одним значительным недостатком является то, что все ветровые турбины имеют несколько сходств в своих кривых мощности, но у них нет формализованного описания, которое могло бы помочь в выборе лучших турбин, соответствующих конкретным участкам (без точных измерений в окрестности скоростей режимов включения и выключения).

Цель исследования. Поскольку ощутимо не хватает математического описания кривых мощности ветровых турбин, целью работы является получение модели таких кривых.

Методика реализации. Кривая мощности состоит из семи частей. Фактические кривые мощности отдаленно напоминают трапеции с криволинейными флангами. Из-за инерции их криволинейность хуже для тех ветровых турбин, мощность которых выше. Поскольку лопасти промышленных ветровых турбин слишком массивны, их инерция создает эффекты запаздывания, что могло бы быть смоделировано с использованием естественной гладкости кривых мощности. Для описания этой гладкости наряду с криволинейностью для флангов мы используем две возрастающие и две убывающие экспоненциальные функции.

Результаты исследования. Функция мощности ветровой турбины состоит из двух нулевых частей, одной части с номинальной мощностью и предложенных четырех экспоненциальных частей. Части режима включения описываются двумя возрастающими экспоненциальными функциями, коэффициенты экспоненциального роста которых равны. Части режима выключения описываются двумя убывающими экспоненциальными функциями, коэффициенты экспоненциального убывания которых также равны. Такие равные коэффициенты обеспечивают сильную дифференцируемость кривой мощности в пределах этих частей.

Выводы. Экспоненциальная модель предназначена для общего описания кривой мощности ветровой турбины. Имея дифференцируемые части на режимах включения и выключения, она предлагает именно то "естественное сглаживание", которое происходит в действительности благодаря высокоинерционным лопастям ветровых турбин. Такая модель может использоваться не обязательно для подгонки некоторых экспериментальных данных, но и для образцового моделирования кривых мощности.

Ключевые слова: ветровая турбина; кривая мощности; скорость на режиме включения; скорость на режиме выключения; естественная гладкость; экспоненциальная кривая.

В.В. Романюк

ЕКСПОНЕНЦІАЛЬНА МОДЕЛЬ КРИВОЇ ПОТУЖНОСТІ ВІТРОВОЇ ТУРБІНИ З ДИФЕРЕНЦІЙОВАНИМИ ЧАСТИНАМИ НА РЕЖИМАХ ВКЛЮЧЕННЯ І ВИКЛЮЧЕННЯ

Проблематика. Головною характеристикою вітрової турбіни є її крива потужності. Збирання даних вимірювань з потужних вітрових турбін є набагато складнішим, ніж вимірювання характеристик вітрових турбін для індивідуального/домашнього використання. Ще один значний недолік полягає у тому, що всі вітрові турбіни мають кілька подібностей у своїх кривих потужності, але у них немає формалізованого опису, який міг би допомогти у виборі кращих турбін, пристосованих до певних ділянок (без точних вимірювань в околі швидкостей режимів включення і виключення).

Мета дослідження. Оскільки відчутно не вистачає математичного опису кривих потужності вітрових турбін, метою роботи є отримання моделі таких кривих.

Методика реалізації. Крива потужності складається із семи частин. Фактичні криві потужності віддалено нагадують трапеції з криволінійними флангами. Через інерцію їх криволінійність гірша для тих вітрових турбін, потужність яких вища. Оскільки лопаті промислових вітрових турбін занадто масивні, їх інерція призводить до ефектів відставання, що можна було б змоделювати з використанням природної гладкості кривих потужності. Для опису цієї гладкості разом зі згаданою криволінійністю для флангів ми використовуємо дві зростаючі та дві спадаючі експоненціальні функції.

Результати дослідження. Функція потужності вітрової турбіни складається з двох нульових частин, однієї частини з номінальною потужністю та чотирьох запропонованих експоненціальних частин. Частини режиму включення описуються двома зростаючими експоненціальними функціями, коефіцієнти експоненціального росту яких рівні. Частини режиму виключення описуються двома спадаючими експоненціальними функціями, коефіцієнти експоненціального спаду яких також рівні. Такі рівні коефіцієнти забезпечують сильну диференційованість кривої потужності в межах цих частин.

Висновки. Експоненціальна модель призначена для загального опису кривої потужності вітрової турбіни. Маючи диференційовані частини на режимах включення і виключення, вона пропонує саме те “природне згладжування”, що відбувається в дійсності завдяки високоінерційним лопатям вітрових турбін. Така модель може використовуватися не обов’язково для підлаштування деяких експериментальних даних, але й для зразкового моделювання кривих потужності.

Ключові слова: вітрова турбіна; крива потужності; швидкість на режимі включення; швидкість на режимі виключення; природна гладкість; експоненціальна крива.

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