

# ІНФОРМАЦІЙНІ ТЕХНОЛОГІЇ, СИСТЕМНИЙ АНАЛІЗ ТА КЕРУВАННЯ

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## PROCESSING UNCERTAINTIES IN MODELING NONSTATIONARY TIME SERIES USING DECISION SUPPORT SYSTEMS

**Background.** Forecasting of nonlinear nonstationary time series (NNTS) is important problem in economics, marketing, industry, ecology and many other branches of science and practical activities. Successful solution of the problem requires development of modern computer based decision support systems (DSS) capable to generate reliable estimates of forecasts in conditions of uncertainty of various type and origin.

**Objective.** The purpose of the research is as follows: development of requirements to the modern DSS and their formal representation; analysis of uncertainty types characteristic for model building and forecasting; selection of techniques for taking into consideration of the uncertainties; and illustration of the system application to solving the problem of forecasts estimation for heteroskedastic NNTS using statistical data.

**Methods.** To reach the objectives stated the following methods were used: systemic approach to statistical data analysis; statistical approach to identification and taking into consideration of possible uncertainties; Kalman filtering techniques; Bayesian programming approach and statistical criteria of model adequacy and quality of forecasts.

**Results.** Formal description of the DSS is provided, and requirements to its development are given; the classes of mathematical methods necessary for DSS implementation are proposed; some approaches to formal taking into consideration of probabilistic, statistical and parametric uncertainties are discussed; and illustrating example of the DSS application is considered.

**Conclusions.** Systemic approach to DSS constructing for solving the problem of nonlinear nonstationary time series forecasting turned out to be very fruitful. Using the system proposed it is possible to take into consideration various uncertainties of probabilistic, statistical and parametric type, and to compute high quality estimates of short and medium term forecasts for NNTS. The approach proposed has good perspectives for the future improvements and enhancement.

**Keywords:** time series forecasting; systemic approach; probabilistic, statistical and parametric uncertainties, decision support system.

### Introduction

Analysis of nonlinear nonstationary time series (NNTS) is an urgent problem not only for financial organizations and companies but for all industrial enterprises, small and medium business, investment and insurance companies etc. Adequate models of NNTS and the forecasts generated with them help to take into consideration a set of various influencing factors and make objective managerial decisions. Another purpose of the studies is in estimating possible risks using forecasts of NNTS volatility. There are several types of NNTS that could be described with mathematical models in the form of appropriately constructed equations or probability distributions. Among them are processes with deterministic and stochastic trends, and heteroskedastic processes. As of today the following mathematical models are widely used for describing dynamics of NNTS: linear and nonlinear regression (logit and probit, polynomials, splines), autoregressive integrated moving average (ARIMA) models, autoregressive conditionally heteroskedastic models

(ARCH), generalized ARCH (GARCH), dynamic Bayesian networks, support vector machine (SVM) approach, neural networks and neuro-fuzzy techniques as well as combinations of the approaches mentioned [1–5].

All types of mathematical modeling usually need to cope with various kinds of uncertainties related to statistical data, structure of the process under study and its model, parameter uncertainty, and uncertainties relevant to the models and forecasts quality. Reasoning and decision making are very often performed with leaving many facts unknown or rather vaguely represented in processing of data and expert estimates. To avoid or to take into consideration the uncertainties and improve this way quality of the final result (processes forecasts and decisions based on them) it is necessary to construct appropriate computer based decision support systems (DSS) for solving multiple specific problems.

Selection and application of a specific model for process description and forecasts estimation depends on application area, availability of statistical

data, qualification of personnel, who work on the data analysis problems, and availability of appropriate applied software. Better results for estimation of processes forecasts is usually achieved with application of ideologically different techniques combined in the frames of one computer system. Such approach to solving the problems of quality forecasts estimation can be implemented in the frames of modern decision support systems (DSS). DSS today is a powerful instrument for supporting user's (managerial) decision making as far as it combines a set of appropriately selected data and expert estimates processing procedures aiming to reach final result of high quality: objective high quality alternatives for a decision making person (DMP). Development of a DSS is based on modern theory and techniques of system analysis, data processing systems, estimation and optimization theories, mathematical and statistical modeling and forecasting, decision making theory as well as many other results of theory and practice of processing data and expert estimates [6–8].

The paper considers the problem of DSS constructing for solving the problems of modeling and estimating forecasts for selected types of NNTS with the possibility for application of alternative data processing techniques, modeling and estimation of parameters and states for the processes under study. One of such widely used parameters in economic and financial sphere is volatility.

### Problem formulation

The purpose of the study is as follows: 1) development of requirements to the modern decision support systems and their formal representation; 2) analysis of uncertainty types characteristic for model building and forecasting; 3) selection of techniques for taking into consideration of the uncertainties; 4) selection of mathematical modeling and forecasting techniques for NNTS – heteroskedastic processes; 5) illustration of the system application to solving selected problem of forecasts estimation for heteroskedastic NNTS using appropriate statistical data.

### Requirements to modern DSS

Modern DSS are rather complex multifunctional (possibly spatially distributed) highly developed information processing computing systems with hierarchical architecture that corresponds to the nature of decision making by a decision making person. To make their performance maximum useful

and convenient for users of different levels (like engineering and managerial staff) they should satisfy some known general requirements. Formally DSS could be defined as follows:

$$DSS = \{DKB, PDP, ST, MSE, \\ MPE, FGP, DQ, MQ, FEQ, AQ\},$$

where *DKB* is data and knowledge base; *PDP* is a set of procedures for preliminary data processing; *ST* is a set of statistical tests for determining possible effects contained in data (like integration or heteroskedasticity); *MSE* is a set of procedures for estimating of mathematical model structure; *MPE* is a set of procedures for estimation of mathematical model parameters; *FGP* are forecasts generating procedures; *DQ, MQ, FEQ, AQ* are the sets of statistical quality criteria for estimating quality of data, models, forecast estimates, and decision alternatives.

Such systems should satisfy the following general requirements that follow from the system analysis principles: 1) contain highly developed bases of data and knowledge: mathematical models constructing procedures; quality criteria for each type of computing, and model selection rules; 2) to achieve high quality of the final result the hierarchy of the system functioning should correspond to the hierarchic process of making decision by a human; 3) their interface should be based on the human factors principles: user friendly, convenient and simple for use, as well as adaptive to users of various levels (e.g., engineering and managerial staff); 4) the system should possess an ability for learning in the process of its functioning, i.e. accumulate appropriate knowledge regarding possibilities of solving the problems of definite class; 5) an active use of artificial intelligence data processing techniques, helping to gradually transform the DSS into intelligent one; 6) the organization aspects and techniques for computing procedures should provide for an appropriate rate of computing that corresponds to the DMP requirements with regard to the rate of generating alternatives and reaching the final result; 7) precision (quality) of computing should satisfy preliminary established requirements by a user and developer; 8) intermediate and final results of computations should be controlled with appropriate sets of analytic quality criteria, what would allow to enhance significantly quality and reliability of the final result (decision alternatives); 9) DSS should generate all necessary for a user formats and types of intermediate and final results representations with taking into consideration the users of various le-

vels; 10) the system should contain the means for exchanging with data and knowledge with other information processing systems via local and/or global computer nets; 11) to make the system functionality complete and flexible DSS should be easily expandable with new functions regarding data processing, results representation and control with appropriate statistical criteria, model constructing and alternatives generating.

Satisfaction of all the requirements mentioned above provides a possibility for effective practical application of the system developed and enhancing general behavioral effect of the DSS as a whole for a specific company or an enterprise within long periods of time.

### Basic mathematical tools for DSS

All mathematical tools and techniques hired for development and implementation of DSS could be divided in the two following groups: 1 – general purpose tools that provide for implementation of the system functions; and 2 – special purpose methods and techniques that are necessary for solving specific problems regarding preliminary and basic data processing, model constructing, alternatives generating, selecting the best alternative for further implementation and forecasting of the implementation consequences.

The group of the general purpose methods includes the following ones:

- data and knowledge collecting and editing procedures;
- preliminary data processing techniques such as digital filtering, normalization, imputation of missing values, detecting special effects such as regime switching, seasonal effects, spikes, nonstationarity etc;
- the methods for accumulating information regarding previous applications of DSS to problem solving for the retrospective analysis and repetitive use;
- computer graphics techniques;
- techniques for syntactic analysis to be used in a command interpreter (language system of DSS);
- methods for setting up necessary communications with other information processing systems via local and global nets;
- logical rules to control the system functioning.

The set of the methods mentioned are to be modified or expanded depending on a specific application.

Selection of the application defined mathematical methods for a DSS functioning depends on

a specific system application area, possible specific problem statements regarding data processing, model building, processes forecasting, and alternatives generating. However, it is possible to state that in most cases of DSS development it is necessary to use the following mathematical methods:

- methods and methodologies for mathematical (statistical and probabilistic) modeling using statistical/experimental data and expert estimates;
- forecasts estimating techniques (including risks estimation) on the basis of the models constructed with possibilities for combining the forecasts computed with different techniques;
- operations research optimization techniques and dynamic optimization (optimal control) methods;
- the methods for forecasting/foresight of decision implementation consequences;
- the sets of specific analytic (statistical) criteria to control the processes of computations performed at each stage of data processing, model constructing and alternatives generation aiming to reach high quality of intermediate and final results.

All the methods and methodologies mentioned are described with necessary completeness in special modern literature. For example, time series modeling and forecasting are presented in many references, more particularly in [9, 10], and financial risks modeling, evaluation and management is considered in a vast literature, say in [11, 12]. The task for a DSS developer is in appropriate selection of model classes, modeling and optimization techniques, quality criteria as well as relevant methodologies for appropriate organization of all computational procedures.

### Coping with uncertainties

As it was mentioned above all types of mathematical modeling usually need to consider various kinds of uncertainties caused by data, informational structure of a process under study and its model, parameter uncertainty, and uncertainties relevant to the quality of models and forecasts. In many cases a researcher has to cope with the following basic types of uncertainties: structural, statistical and parametric. Structural uncertainties are encountered in the cases when structure of the process under study (and respectively its model) is unknown or not clearly enough defined (known partially). For example, when the functional approach to model constructing is applied usually we do not know object (or a process) structure, it is estimated with appropriate model structure estimation techniques: cor-

relation analysis, estimation of mutual information, lags estimation, testing for nonlinearities and non-stationarity, identification of external disturbances type etc. The sequence of actions necessary for identification, processing and taking into consideration of uncertainties could be formulated as follows:

- identification and reduction of data uncertainty;
- model structure and parameters estimation;
- reduction of uncertainties related to the model structure and parameters estimation;
- estimation of forecasts and reduction of respective uncertainties;
- selection of the best final result.

All the tasks mentioned above are solved sequentially (or in an adaptive loop) with appropriately designed and implemented DSS.

We consider uncertainties as the factors that influence negatively the whole process of mathematical model constructing, forecasts and possible risk estimating and generating of alternative decisions. They are inherent to the process being studied due to incomplete or noise corrupted data, complex stochastic external influences, incompleteness or inexactness of our knowledge regarding the objects (systems) structure, incorrect application of computational procedures etc. The uncertainties very often appear due to incompleteness of data, noisy measurements or they are invoked by sophisticated stochastic external disturbances with complex unknown probability distributions, poor estimates of model structure or by a wrong selection of parameter estimation procedure. The problem of uncertainties identification is solved with application of special statistical tests and visual studying of available data.

As far as we usually work with stochastic data, correct application of existing statistical techniques provides a possibility for approximate estimation of a system (and its model) structure. To find “the best” model structure it is recommended to apply adaptive estimation schemes that provide automatic search in a wide range of model structures and parameters (model order, time lags, and possible nonlinearities). It is often possible to perform the search in the class of regression type models with the use of information criterion of the following type [2]:

$$N \log(FPE) = N \log(V_N(\hat{\theta})) + N \log\left(\frac{N+p}{N-p}\right), \quad (1)$$

where  $\hat{\theta}$  is a vector of model parameters estimates;  $N$  is a power of time series used;  $FPE$  is final prediction error term;  $V_N(\hat{\theta})$  is determined by the sum of squared errors;  $p$  is a number of model parame-

ters. The value of the criteria (1) is asymptotically equivalent to the Akaike information criterion as  $N \rightarrow \infty$ . As the amount of data,  $N$ , may be limited, then an alternative, the minimum description length (MDL) criterion

$$MDL = \log(V_N(\hat{\theta})) + p \frac{\log(N)}{N}$$

could be used to find the model that adequately represents available data with the minimum amount of information.

There are several possibilities for adaptive model structure estimation: (1) application of statistical criteria for detecting possible nonlinearities and the type of nonstationarity (integrated or heteroskedastic process); (2) analysis of partial autocorrelation for determining autoregression order; (3) automatic estimation of the exogenous variable lag (detection of leading indicators); (4) automatic analysis of residual properties; (5) analysis of data distribution type and its use for selecting correct model estimation method; (6) adaptive model parameter estimation with hiring extra data; (7) optimal selection of weighting coefficients for exponential smoothing, nearest neighbor and other techniques. The use of a specific adaptation scheme depends on volume and quality of data, specific problem statement, requirements to forecast estimates etc.

The adaptive estimation schemes also help to cope with the model parameters uncertainties. New data are used to re-compute model parameter estimates that correspond to possible changes in the object under study. In the cases when model is nonlinear alternative parameter estimation techniques (say, MCMC) could be hired to compute alternative (though admissible) sets of parameters and to select the most suitable of them using statistical quality criteria.

#### ***Processing some types of stochastic uncertainties.***

While performing practical modeling very often statistical characteristics (covariance matrix) of stochastic external disturbances and measurement noise (errors) are unknown. To eliminate this uncertainty optimal filtering algorithms are usually applied that provide for a possibility of simultaneous estimation of object (system) states and the covariance matrices. One of the possibilities to solve the problem is optimal Kalman filter. Kalman filter is used to find optimal estimates of system states on the bases of the system model represented in a convenient state space form as follows:

$$\mathbf{x}(k) = \mathbf{A}(k, k-1) \mathbf{x}(k-1) + \mathbf{B}(k, k-1) \mathbf{u}(k-1) + \mathbf{w}(k), \quad (2)$$

where  $\mathbf{x}(k)$  is  $n$ -dimensional vector of system states;  $k = 0, 1, 2, \dots$  is discrete time;  $\mathbf{u}(k-1)$  is  $m$ -dimensional vector of deterministic control variables;  $\mathbf{w}(k)$  is  $n$ -dimensional vector of external random disturbances;  $\mathbf{A}(k, k-1)$  is  $(n \times n)$ -matrix of system dynamics;  $\mathbf{B}(k, k-1)$  is  $(n \times m)$ -matrix of control coefficients. The double argument  $(k, k-1)$  means that the variable or parameter is used at the moment  $k$ , but its value is based on the former (earlier) data processing including moment  $(k-1)$ . Usually the matrices  $\mathbf{A}$  and  $\mathbf{B}$  are written with one argument like  $\mathbf{A}(k)$ , and  $\mathbf{B}(k)$ , to simplify the text. Obviously stationary system model is described with constant parameters like  $\mathbf{A}$ , and  $\mathbf{B}$ . As far as matrix  $\mathbf{A}$  is a link between two consequent system states, it is also called state transition matrix. Discrete time  $k$  and continuous time  $t$  are linked to each other via data sampling time  $T_s$ :  $t = k T_s$ . In the classic problem statement for optimal filtering the vector sequence of external disturbances  $\mathbf{w}(k)$  is supposed to be zero mean white Gaussian noise with covariance matrix  $\mathbf{Q}$ , i.e. the noise statistics are as follows:

$$E[\mathbf{w}(k)] = 0 \quad \forall k; \quad E[\mathbf{w}(k)\mathbf{w}^T(j)] = \mathbf{Q}(k)\delta_{kj},$$

where  $\delta_{kj}$  is Kronecker delta-function:  $\delta_{kj} = \begin{cases} 0, & k \neq j \\ 1, & k = j \end{cases}$ ;

$\mathbf{Q}(k)$  is positively defined covariance  $(n \times n)$ -matrix. The diagonal elements of the matrix are variances for the components of disturbance vector  $\mathbf{w}(k)$ . Initial system state  $\mathbf{x}_0$  is supposed to be known and the measurement equation for vector  $\mathbf{z}(k)$  of output variables is described by the equation:

$$\mathbf{z}(k) = \mathbf{H}(k)\mathbf{x}(k) + \mathbf{v}(k), \quad (3)$$

where  $\mathbf{H}(k)$  is  $(r \times n)$  observation (coefficients) matrix;  $\mathbf{v}(k)$  is  $r$ -dimensional vector of measurement noise with statistics:

$$E[\mathbf{v}(k)] = 0, \quad E[\mathbf{v}(k)\mathbf{v}^T(j)] = \mathbf{R}(k)\delta_{kj},$$

where  $\mathbf{R}(k)$  is  $(r \times r)$  positively defined measurement noise covariance matrix, the diagonal elements of which represent variances of additive noise for each measurable variable. The noise of measurements is also supposed to be zero mean white noise sequence that is not correlated with external disturbance  $\mathbf{w}(k)$  and initial system state. For the system (2), (3) with state vector  $\mathbf{x}(k)$  it is necessary to find optimal state estimate  $\hat{\mathbf{x}}(k)$  at arbitrary moment  $k$  as a linear combination of estimate

$\mathbf{x}(k-1)$  at the previous moment  $(k-1)$  and the last measurement available,  $\mathbf{z}(k)$ . The estimate of state vector  $\hat{\mathbf{x}}(k)$  is computed as optimal one with minimizing the expectation of the sum of squared errors, i.e.:

$$E[(\hat{\mathbf{x}}(k) - \mathbf{x}(k))^T(\hat{\mathbf{x}}(k) - \mathbf{x}(k))] = \min_{\mathbf{K}}, \quad (4)$$

where  $\mathbf{x}(k)$  is an exact value of state vector that can be found as deterministic part of the state equation (2);  $\mathbf{K}$  is optimal matrix gain that is determined as a result of minimizing quadratic criterion (4).

Thus, the filter is constructed to compute optimal state vector  $\hat{\mathbf{x}}(k)$  in conditions of influence of external random system disturbances and measurement noise. Here uncertainty arises when we don't know estimates of covariance matrices  $\mathbf{Q}$  and  $\mathbf{R}$ . To solve the problem an adaptive Kalman filter is to be constructed that allows to compute estimates of  $\hat{\mathbf{Q}}$  and  $\hat{\mathbf{R}}$  simultaneously with the state vector  $\hat{\mathbf{x}}(k)$ . Another choice is in constructing separate algorithm for computing the values of  $\hat{\mathbf{Q}}$  and  $\hat{\mathbf{R}}$ . A convenient statistical algorithm for estimating the covariance matrices was proposed [11]:

$$\hat{\mathbf{R}} = \frac{1}{2} [\hat{\mathbf{B}}_1 + \mathbf{A}^{-1}(\hat{\mathbf{B}}_1 - \hat{\mathbf{B}}_2)(\mathbf{A}^{-1})^T],$$

$$\hat{\mathbf{Q}} = \hat{\mathbf{B}}_1 - \hat{\mathbf{R}} - \mathbf{A}\hat{\mathbf{R}}\mathbf{A}^T,$$

where  $\hat{\mathbf{B}}_1 = E\{[\mathbf{z}(k) - \mathbf{A}\mathbf{z}(k-1)][\mathbf{z}(k) - \mathbf{A}\mathbf{z}(k-1)]^T\}$ ;  
 $\hat{\mathbf{B}}_2 = E\{[\mathbf{z}(k) - \mathbf{A}^2\mathbf{z}(k-2)][\mathbf{z}(k) - \mathbf{A}^2\mathbf{z}(k-2)]^T\}$ .

The matrices  $\hat{\mathbf{Q}}$  and  $\hat{\mathbf{R}}$  are used in the optimal filtering procedure as follows:

$$\mathbf{S}(k) = \mathbf{A}\mathbf{P}(k-1)\mathbf{A}^T + \hat{\mathbf{Q}}; \quad \Delta(k) = \mathbf{S}(k)[\mathbf{S}(k) + \hat{\mathbf{R}}]^\#,$$

$$\mathbf{P}(k) = [\mathbf{I} - \Delta(k)]\mathbf{S}(k), \quad k = 0, 1, 2, \dots,$$

where  $\mathbf{S}(k)$  and  $\mathbf{P}(k)$  are prior and posterior covariance matrices of estimates errors respectively; the symbol “#” denotes pseudo-inverse;  $\mathbf{A}^T$  means matrix transposition;  $\Delta(k)$  is a matrix of intermediate covariance results. The algorithm was successfully applied to the covariances estimating in many practical applications. The computation experiments showed that the values of  $\Delta(k)$  become stationary after about 20–25 periods of time (sampling periods) in a scalar case, though this figure is growing substantially with the growth of dimensionality of the system under study. It was also determined that the parameter estimators are very sensitive to the initial conditions of the system. The initial condi-

tions should differ from zero enough to provide stability for the estimates generated.

Other appropriate instruments for taking into consideration the uncertainties are fuzzy logic, neuro-fuzzy models, Bayesian networks, appropriate types of distributions etc. Some of statistical data uncertainties such as missing measurements, extreme values and high level jumps of stochastic origin could be processed with appropriately selected statistical procedures. There exists a number of data imputation schemes that help to complete the sets of the data collected. For example, very often missing measurements for time series could be generated with appropriately selected distributions or in the form of short term forecasts. Appropriate processing of jumps and extreme values helps with adjusting data non-stationarity and to estimate correctly the probability distribution for the stochastic processes under study.

**Processing data with missing observations (data are in the form of time series).** As of today for the data in the time series form the most suitable imputation techniques are as follows: simple averaging when it is possible (when only a few values are missing); generation of forecast estimates with the model constructed using available measurements; generation of missing estimates from distributions the form and parameters of which are again determined using available part of data and expert estimates; the use of optimization techniques, say appropriate forms of EM-algorithms (expectation maximization); exponential smoothing etc. It should also be mentioned that optimal Kalman filter can also be used for imputation of missing data because it contains "internal" forecasting function that provides a possibility for generating quality short-term forecasts [12]. Besides, it has a feature of fusion the data coming from alternative sources and improving this way the quality of state vector and its forecasts.

Further reduction of this uncertainty is possible thanks to application of several forecasting techniques to the same problem with subsequent combining of separate forecasts using appropriate weighting coefficients. The best results of combining the forecasts is achieved when variances of forecasting errors for different forecasting techniques do not differ substantially (at any rate the orders of the variances should be the same).

**Coping with uncertainties of model parameters estimates.** Usually uncertainties of model parameter estimates such as bias and inconsistency result from low informative data, or data do not correspond to normal distribution, what is required in a case of LS application for parameter estimation. This situation

may also take place in a case of multicollinearity of independent variables and substantial influence of process nonlinearity that for some reason has not been taken into account when model was constructed. When power of the data sample is not satisfactory for model construction it could be expanded by applying special techniques, or simulation is hired, or special model building techniques, such as group method for data handling (GMDH), are applied. Very often GMDH produces results of acceptable quality with rather short samples. If data do not correspond to normal distribution, then ML technique could be used or appropriate Monte Carlo procedures for Markov Chains (MCMC) [13]. The last techniques could be applied with quite acceptable computational expenses when the number of parameters is not large.

**Dealing with model structure uncertainties.**

When considering mathematical models it is convenient to use proposed here a unified notion of a model structure which we define as follows:  $S = \{r, p, m, n, d, w, l\}$ , where  $r$  is model dimensionality (number of equations);  $p$  is model order (maximum order of differential or difference equation in a model);  $m$  is a number of independent variables in the right hand side of a model;  $n$  is a nonlinearity and its type;  $d$  is a lag or output reaction delay time;  $w$  is stochastic external disturbance and its type;  $l$  are possible restrictions for the variables and/or parameters. When using DSS, the model structure should practically always be estimated using data. It means that elements of the model structure accept almost always only approximate values. When a model is constructed for forecasting we build several candidates and select the best one of them with the set of model quality statistics. Generally we could define the following techniques to fight structural uncertainties: gradual improvement of model order (AR( $p$ ) or ARMA ( $p, q$ )) applying adaptive approach to modeling and automatic search for the "best" structure using complex statistical quality criteria; adaptive estimation (improvement) of input delay time (lag) and data distribution type with its parameters; describing detected process nonlinearities with alternative analytical forms with subsequent estimation of model adequacy and forecast quality. As another example of complex statistical model adequacy and forecast quality criterion could be the following:

$$J = |1 - R^2| + \alpha \ln \left[ \sum_{k=1}^N e^2(k) \right] + |2 - DW| + \beta \ln(1 + MAPE) + U \rightarrow \min_{\theta_i}$$

where  $R^2$  is a determination coefficient;  $DW$  is Durbin-Watson statistic;  $MAPE$  is mean absolute percentage error for forecasts;  $\sum_{k=1}^N e^2(k) = \sum_{k=1}^N [y(k) - \hat{y}(k)]^2$  is a sum of squared model errors;  $U$  is Theil coefficient that measures forecasting characteristic of a model;  $\alpha, \beta$  are appropriately selected weighting coefficients;  $\hat{\theta}_i$  is parameter vector for the  $i$ -th candidate model. A criterion of this type is used for automatic selection of the best candidate model. The criterion also allows operation of DSS in the automatic adaptive mode. Obviously, other forms of the complex criteria are possible. While constructing the criterion it is important not to overweigh separate members in the right hand side.

**Coping with uncertainties of a level (amplitude) type.** The use of random (i.e. with random amplitude or a level) and/or non-measurable variables leads to necessity of hiring fuzzy sets for describing such situations. The variable with random amplitude can be described with some probability distribution if the measurements are available or they come for analysis in acceptable time span. However, some variables cannot be measured (registered) in principle, say amount of shadow capital that "disappears" every month in offshore, or amount of shadow salaries paid at some company, or a technology parameter that cannot be measured on-line due to absence of appropriate gauge. In such situations we could assign to the variable a set of possible values in the linguistic form as follows: *capital amount* = {very low, low, medium, high, very high}. There exists a complete necessary set of mathematical operations to be applied to such fuzzy variables. Finally, fuzzy value could be transformed into usual "exact" form using known techniques.

**Processing probabilistic uncertainties.** To fight probabilistic uncertainties it is possible to hire Bayesian approach that helps to construct models in the form of conditional distributions for the sets of random variables. Usually such models represent the process (under study) variables themselves, stochastic disturbances and measurement errors or noise. The problem of distribution type identification also arises in regression modeling. Each probability distribution is characterized by a set of specific values that random variable could take and the probabilities for these values. The problem is in the distribution type identification and estimating its param-

eters. The probabilistic uncertainty (will some event happen or not) could be solved with various models of Bayesian type. This approach is known as Bayesian programming or paradigm. The generalized structure of the Bayesian program includes the following steps: (1) problem description and statement with putting the question regarding estimation of conditional probability in the form:  $p(X_i|D, Kn)$ , where  $X_i$  is the main (goal) variable or event; the probability  $p$  should be found as a result of application of some probabilistic inference procedure; (2) statistical (experimental) data  $D$  and knowledge  $Kn$  are to be used for estimating model and parameters of specific type; (3) selected and applied probabilistic inference technique should give an answer to the question put above; (4) analysis of quality of the final result. The steps given above are to some extent "standard" regarding model constructing and computing probabilistic inference using statistical data available. This sequence of actions is naturally consistent with the methods of cyclic structural and parametric model adaptation to the new data and operating modes (and possibly expert estimates).

One of the most popular Bayesian approaches today is created by the models in the form of static and dynamic Bayesian networks (BN). Bayesian networks are probabilistic and statistical models represented in the form of directed acyclic graphs (DAG) with vertices as variables of an object (system) under study, and the arcs showing existing causal relations between the variables. Each variable of BN is characterized with complete finite set of mutually excluding states. Formally BN could be represented with the four following components:  $\mathbf{N} = \langle \mathbf{V}, \mathbf{G}, \mathbf{P}, \mathbf{T} \rangle$ , where  $\mathbf{V}$  stands for the set of model variables;  $\mathbf{G}$  represents directed acyclic graph;  $\mathbf{P}$  is joint distribution of probabilities for the graph variables (vertices),  $\mathbf{V} = \{X_1, \dots, X_n\}$ ; and  $\mathbf{T}$  denotes conditional and unconditional probability tables for the graphical model variables [14, 15]. The relations between the variables are established via expert estimates or applying special statistical and probabilistic tests to statistical data (when available) characterizing dynamics of the variables.

The process of constructing BN is generally the same as for models of other types, say regression models. The set of the model variables should satisfy the Markov condition that each variable of the network does not depend on all other variables but for the variable's parents. In the process of BN constructing first the problem is solved of compu-

ting mutual information values between all variables of the net. Then an optimal BN structure is searched using acceptable quality criterion, say well-known minimum description length (MDL) that allows analyzing and improving the graph (model) structure at each iteration of the learning algorithm applied. Bayesian networks provide the following advantages for modeling: the model may include qualitative and quantitative variables simultaneously as well as discrete and continuous ones; number of the variables could be very large (thousands); the values for conditional probability tables could be computed with the use of statistical data and expert estimates; the methodology of BN constructing is directed towards identification of actual causal relations between the variables hired what results in high adequacy of the model; the model is also operable in conditions of missing data.

To reduce an influence of probabilistic and statistical uncertainties on models quality and the forecasts based upon them it is also possible to use the models in the form of Bayesian regression based on analysis of actual distributions, of model variables and parameters. Consider a simple two variables regression

$$y(k)|x(k) = \beta_1 + \beta_2 x(k) + u(k), \quad k=0,1,\dots,n.$$

It is supposed that of random values  $u_1, \dots, u_n$  are independent and belong, for example, to normal distribution,  $\{u(k)\} \sim N(0, \sigma_u^2)$ ; here vector of unknown parameters includes three elements,  $\theta = (\beta_1, \beta_2, \sigma_u)^T$ . The likelihood function for dependent variable  $\mathbf{y} = (y_1, \dots, y_n)^T$  and predictor  $\mathbf{x} = (x_1, \dots, x_n)^T$  without proportion coefficient is determined as follows:

$$L(\mathbf{y}|\mathbf{x}, \beta_1, \beta_2, \sigma_u) = \frac{1}{\sigma_u^N} \exp \left\{ -\frac{1}{2\sigma_u^2} \sum_{k=1}^N [y(k) - \beta_1 - \beta_2 x(k)]^2 \right\}.$$

Using simplified (non-informative) distributions for the model parameters

$$g(\beta_1, \beta_2, \sigma_u) = g_1(\beta_1) g_2(\beta_2) g_3(\sigma_u),$$

$$g_1(\beta_1) \propto \text{const},$$

$$g_2(\beta_2) \propto \text{const},$$

$$g_3(\sigma_u) \propto 1/\sigma_u,$$

and Bayes theorem it is possible to find joint posterior distribution for the parameters in the form [16]:

$$h(\beta_1, \beta_2, \sigma_u | \mathbf{x}, \mathbf{y}) \propto \frac{1}{\sigma} \frac{1}{\sigma^N} \exp \left[ -\frac{1}{2\sigma^2} \sum_{k=1}^N (y(k) - \beta_1 - \beta_2 x(k))^2 \right], \quad -\infty < \beta_1, \beta_2 < +\infty, \quad 0 < \sigma_u < \infty.$$

Maximum likelihood estimates for the model parameters are determined as follows:

$$\begin{aligned} \hat{\beta}_1 &= \bar{y} - \hat{\beta}_2 \bar{x}; \\ \hat{\beta}_2 &= \frac{\sum_{k=1}^N [x(k) - \bar{x}][y(k) - \bar{y}]}{\sum_{k=1}^N [x(k) - \bar{x}] \sum_{k=1}^N [y(k) - \bar{y}]} \end{aligned}$$

where  $\bar{x} = N^{-1} \sum_{k=1}^N x(k)$ ,  $\bar{y} = N^{-1} \sum_{k=1}^N y(k)$ , with unbiased sample estimate of variance:

$$\hat{\sigma}_u^2 = s^2 = \frac{1}{N-2} \sum_{k=1}^N [y(k) - \hat{\beta}_1 - \hat{\beta}_2 x(k)]^2.$$

Joint posterior density for the model parameters corresponds to two dimensional Student distribution:

$$\begin{aligned} h_1(\beta_1, \beta_2 | \mathbf{y}, \mathbf{x}) &\propto \{(N-2)s^2 + N(\beta_1 - \hat{\beta}_1)^2 \\ &+ (\beta_2 - \hat{\beta}_2)^2 \sum_{k=1}^N x(k)^2 \\ &+ 2(\beta_1 - \hat{\beta}_1)(\beta_2 - \hat{\beta}_2) \sum_{k=1}^N x(k)\}^{-0.5N}. \end{aligned}$$

This way we get a possibility for using more exact distributions of model variables and parameters what helps to enhance model quality. Using new observation  $x^*$  and prior information regarding particular model it is possible to determine the forecast interval for the dependent variable,  $y^*$ , as follows:

$$\begin{aligned} p(y^* | x^*) &= \iiint L(y^* | x^*, \beta_1, \beta_2, \sigma) h(\beta_1, \beta_2, \sigma) | \mathbf{x}, \mathbf{y} d\beta_1, d\beta_2, d\sigma. \end{aligned}$$

Another useful Bayesian approach is in hierarchical modeling that is based on a set of simple conditional distributions comprising one model. The approach is naturally combined with the theory of computing Bayesian probabilistic inference using modern computational procedures [17]. The hierarchical models belong to the class of marginal models where the final result is provided in the form of a distribution  $P(\mathbf{y})$ , where  $\mathbf{y}$  is available data vector. The models are formed from the sequence of conditional distributions for selected variables including the hidden ones. The hierarchical representation of parameters usually supposes that data,  $\mathbf{y}$ , is situ-



ated at the lower (first) level, model parameters (second level)  $\theta = (\theta_i, i=1, 2, \dots, n)$ ,  $\theta_i \sim N(\mu, \tau^2)$ , determine distributions of dependent variables  $y_i \sim N(\theta_i, \sigma^2)$ ,  $i=1, 2, \dots, n$ , and parameters  $\{\theta_i\}$  are determined by the pair,  $(\mu, \tau^2)$ , of the third level. Supposing the parameters  $\sigma^2$  and  $\tau^2$  accept known finite values, and parameter  $\mu$  is unknown with the prior  $\pi_\mu$ , then joint prior density for  $(\theta, \mu)$  could be presented in the form:  $\pi_\mu(\mu) \prod_i \pi_{\theta_i}(\theta_i | \mu)$ , and the prior for parameter vector  $\theta$  will be defined by the integral:  $p(\theta) = \int \pi_\mu(\mu) \prod_i \pi_{\theta_i}(\theta_i | \mu) d\mu$ .

### Generation and implementation of alternatives with the DSS

Decision making process includes rather sophisticated procedures that could be partially or completely iterative, i.e. executed repeatedly when the alternative found is not satisfactory for a decision making person. DSS could return automatically (or on DMP initiative) to the previous stages of available data and knowledge analysis.

The whole process of making and implementing decisions could be considered as consisting of the stages given below.

1 – A thorough analysis of the decision problem using all available sources of information, collection of data and knowledge relevant to the problem. At this stage it is also important to consider and use former solutions to the problem if such are available. The information regarding former solutions of similar problem could be helpful for correct problem statement, to select appropriate techniques for data analysis, to speed up alternatives generation, and to decline the alternatives that turned out to be ineffective in the past.

2 – Selection of a class (classes) of mathematical models for the problem description, and analysis of the possibility for the use of available (previously developed) models. The models could belong to different classes as far as they can be formulated in continuous or discrete time, be linear or nonlinear, they could be developed according to the structural or functional approach etc. In some cases it is necessary to construct complex simulative model that would include a set of simpler models of different classes.

3 – Development of new models for the problem (process, object, system) under study what includes structure and parameter estimation for can-

didate models using available data (and possibly expert estimates) and knowledge of various types. The alternative structures of candidate models provide a possibility for selecting the best one of them for generating alternative decisions (loss estimates, forecasts, probability of risk estimates, control actions etc) on their bases.

4 – Analysis of the candidate models constructed and selecting the best one of them with application of a set of statistical quality criteria and expert estimates. At this stage again more than one model could be selected for the further use as far as the best model (for a particular application) can be found only after application of the candidates for solving particular problem, i.e. after alternatives generating and estimating possible consequences of their implementation.

5 – Application of the model (models) selected for solving the problem of risk estimation and/or control (or management) problem (when necessary). If the forecasts or controls computed are not satisfactory we should return back to the stage one or stage three, and repeat the process of model constructing. At this stage another set of statistical quality criteria should be applied to the analysis of risk estimates, forecasts or controls.

6 – Generating a set of alternatives with the use of the model (models) constructed and various admissible initial conditions and constraints on variables. In a case of controls generating the alternatives could be built with different optimality criteria, utility functions or other criteria.

7 – Analysis of the alternatives generated with the experts of an enterprise or a company, and final selection of the best one for practical implementation. In a case when no alternative is acceptable we should return back to the model constructing or alternative generating stages. New knowledge or data could be required for the next iteration of computing new decision alternatives.

8 – Planning of actions and estimation of financial, material and human resources that are necessary for implementation of the alternative selected. Determining of the time horizon (horizon of control) necessary for implementing the decision made.

9 – Implementation of the decision made: current monitoring of availability and spending the necessary resources, estimation of necessary time frames, registering and quality estimation of intermediate and final results.

10 – Application of possible analytic and expert quality criteria to estimation of final results.

11 – Analysis of the final results by the company experts, and final elucidation of advantages

and disadvantages of the alternative implemented; analysis of the decision making and implementing process, and forming forecasts (foresights) for the future.

12 – Writing the final report on the tasks performed.

### Data, model and forecasts quality criteria

To achieve reliable high quality final result of risk estimation and forecasting at each stage of computational hierarchy separate sets of statistical quality criteria have been used. Data quality control is performed with the following criteria:

- database analysis for missing values using developed logical rules, and imputation of missed values with appropriately selected techniques;
- analysis of data for availability of outliers with special statistical tests, and processing of outliers to reduce their negative influence on statistical properties of the data available;
- normalizing of data in the selected range in a case of necessity;
- application of low-order digital filters (usually low-pass filters) for separation of observations from measurement noise;
- application of optimal (usually Kalman) filters for optimal state estimation and fighting stochastic uncertainties;
- application of principal component method to achieve desirable level of orthogonalization between the variables selected;
- computing extra indicators for the use in regression and other models (say, moving average processes based upon measurements of dependent variables).

It is also useful to test how informative is the data collected. Very formal indicator for the data being informative is its sample variance. It is considered formally that the higher is the variance the richer is the data with information. Another criterion is based on computing derivatives with a polynomial that describes data in the form of a time series. For example, the equation given below can describe rather complex process with nonlinear trend and short-term variations imposed on the trend curve:

$$y(k) = a_0 + \sum_{i=1}^p a_i y(k-i) + c_1 k + c_2 k^2 + \dots + c_m k^m + \varepsilon(k),$$

where  $y(k)$  is basic dependent variable;  $a_i, c_i$  are model parameters;  $k=0,1,2,\dots$  is discrete time;  $\varepsilon(k)$

is a random process that integrates the influence of external disturbances to the process being modeled as well as model structure and parameters errors. Autoregressive part of the model describes the deviations that are imposed on a trend, and the trend itself is described with the  $m$ -th order polynomial of discrete time  $k$ . In this case maximum number of derivatives could be  $m$ , though in practice actual number of derivatives is defined by the largest number  $i$  of parameter  $c_i$ , that is statistically significant. To select the best model constructed the following statistical criteria are used: determination coefficient ( $R^2$ ); Durbin-Watson statistic ( $DW$ ); Fisher  $F$ -statistic; Akaike information criterion ( $AIC$ ), and residual sum of squares ( $SSE$ ). The forecasts quality is estimated with hiring the criteria mentioned in [1, 2]. To perform automatic model selection the above mentioned combined criteria (1) could be hired. The power of the criterion was tested experimentally and proved with a wide set of models and statistical data. Thus, the three sets of quality criteria are used to insure high quality of final result.

### Example of the DSS application

One of the forecasting problems solved with the DSS proposed was estimation of stock prices forecasts with relation to the given level set by the constant  $c$ . The problems of this type are often solved when the stock trading operations are performed. The models constructed for the purpose and the forecasting results are given in Table. Three lower rows of the table characterize the results of application of dynamic Bayesian networks.

The forecasting results achieved with the dynamic Bayesian network were compared to the logistic regression combined with the multiple linear regression:

$$g_{\min}(x_1) = \frac{e^{x_1(k)}}{1 + e^{x_1(k)}},$$

$$x_1(k) = -0.626 - 0.424 \cdot \widehat{S}2(k) - 0.616 \cdot \widehat{P}(k) - 0,81 \cdot \widehat{R}2(k) + 0.773 \cdot \widehat{R}3(k) + 1.739 \cdot yf(k),$$

where  $\widehat{S}2(k), \widehat{P}(k), \widehat{R}2(k), \widehat{R}3(k)$  are technical analysis indicators;  $yf(k)$  is the variable characterizing output of multiple regression, that accepts the value 1 in a case of growing price, and 0 in a case of falling price. Thus, the best price forecasting model in this case turned out to be logistic regression with Backward Selection of independent variables but for

the indicator  $yf(k)$ , which is a forecast computed with multiple regression ( $p = 0.869$ ). The best results of forecasting with DBN were achieved with the "memory depth" value of 5 and the use of linear Kalman filter ( $p = 0.871$ ). It should be noted that computational expenses in latter case were much higher than in the case of logistic regression application. It was also established that the forecasts quality is dependent on the value of the threshold  $c$ , selected as a basis for forecasting.

**Table.** Results of modeling and forecasting

| Threshold value $c$ | The best model | Threshold value for probability | Probability of forecast coincidence with the true direction ( $p$ ) |
|---------------------|----------------|---------------------------------|---|
| 0.0075              | LR (BS) + MR   | 0.47                            | 0.869   |
| 0.0065              | LR (FS) + MR   | 0.5                             | 0.861   |
| 0.0060              | LR (BS) + MR   | 0.5                             | 0.846   |
| 0.0055              | DT (CHAID)     | 0.45                            | 0.832   |
| 0.0050              | LR (FS) + MR   | 0.52                            | 0.831   |
| 0.0045              | LR (BS) + MR   | 0.52                            | 0.828   |
| 0.0040              | LR (BS) + MR   | 0.43                            | 0.826   |
| 0.0035              | LR (BS) + MR   | 0.49                            | 0.822   |
| 0.0010              | LR (FS) + MR   | 0.34                            | 0.732   |
| 0.0005              | LR (FS) + MR   | 0.4                             | 0.710   |
| -0.0020             | LR (BS) + MR   | 0.43                            | 0.677   |
| -0.0025             | LR (BS) + MR   | 0.47                            | 0.699   |
| 0.0075              | DBN-3          | 0.52                            | 0.729   |
| 0.0075              | DBN-3 + KF     | 0.52                            | 0.837   |
| 0.0075              | DBN-5 + KF     | 0.52                            | 0.871   |

*Abbreviations in the table:* LR – logistic regression; MR – multiple regression; DT – decision tree; DBN – dynamic Bayesian network; KF – Kalman filter; FS – forward selection; BS – backward selection; CHAID – Chi-squared Automatic Interaction Detector.

## Conclusions

The general methodology was proposed for constructing DSS for mathematical modeling and forecasting of economic and financial processes that is based on the system analysis principles. As instrumentation for fighting possible structural, statistic and parametric uncertainties the following tech-

niques are used: Kalman filter, various missing data imputation techniques, multiple methods for model parameter estimation, and Bayesian programming approach.

The system proposed has a modular architecture that provides a possibility for easy extension of its functional possibilities with new parameter estimation techniques, forecasting methods, financial risk estimation procedures, and alternatives generation. High quality of the final result is achieved thanks to appropriate tracking of the whole computational processes at all stages of data processing: preliminary data processing, model structure and parameter estimation, computing of short- and middle-term forecasts as well as thanks to convenient for a user intermediate and final results representation. The system is based on the ideologically different techniques of modeling and forecasting what creates a convenient basis for combination of various approaches to achieve the best results. The examples of the system application show that it could be used successfully for solving practical problems of nonlinear nonstationary processes forecasting. The results of computing experiments lead to the conclusion that today nonlinear regression, Bayesian networks and the models resulted from application of regression analysis are quite acceptable instruments for short-term forecasting. It also should be stressed that the DSS constructed turned out to be very useful instrument for a decision maker that helps to perform quality processing of statistical data using different techniques, generate alternatives and to select the best one with a set of appropriate quality criteria. The system performs tracking of the whole computational process using separate sets of statistical quality criteria at each stage of decision making: quality of data, models and forecasts or risk estimates.

The DSS proposed could be used for support of decision making in various areas of scientific and practical activities including strategy development for industrial and financial enterprises, investment companies etc. Further extension of the system functions is planned with new forecasting techniques based on probabilistic approach, fuzzy sets, neuro-fuzzy models.

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## ОБРОБКА НЕВИЗНАЧЕНОСТЕЙ ПРИ МОДЕЛЮВАННІ НЕСТАЦІОНАРНИХ ЧАСОВИХ РЯДІВ З ВИКОРИСТАННЯМ СИСТЕМ ПІДТРИМКИ РІШЕНЬ

**Проблематика.** Прогнозування нелінійних нестационарних часових рядів (ННЧР) – важлива задача для економіки, маркетингу, промисловості, екології та багатьох інших галузей науки і практичної діяльності. Для успішного розв'язання цієї задачі необхідно розробляти сучасні комп'ютерні системи підтримки прийняття рішень (СППР), які дадуть можливість отримувати надійні оцінки прогнозів в умовах наявності невизначеностей різних типів і природи.

**Мета дослідження.** Розробка вимог до сучасних СППР та їх формальне подання; аналіз типів невизначеностей, характерних для процесів побудови математичних моделей та прогнозування; вибір методів для врахування можливих невизначеностей; ілюстрація застосування системи до розв'язання задачі прогнозування гетероскедастичних ННЧР на основі статистичних даних.

**Методика реалізації.** Для досягнення поставленої мети використано такі методи: системний підхід до аналізу статистичних даних; статистичний підхід до ідентифікації та врахування можливих невизначеностей; алгоритми калмановської фільтрації; підхід на основі байєсівського програмування та множини статистичних критеріїв адекватності моделей і якості прогнозів.

**Результати дослідження.** Запропоновано формальний опис СППР та вимоги до їх розробки; встановлено класи математичних методів, які необхідні для успішної реалізації СППР; запропоновано деякі підходи до формального врахування ймовірнісних, статистичних і параметричних невизначеностей і подано ілюстративний приклад застосування СППР.

**Висновки.** Системний підхід до створення СППР для розв'язання задач прогнозування нелінійних нестационарних часових рядів дає можливість успішно розв'язати поставлену задачу. Використовуючи запропоновану систему, можна враховувати невизначеності ймовірнісного, статистичного і параметричного типів і обчислювати високоякісні оцінки коротко- та середньострокових прогнозів ННЧР. Запропонований підхід має хороші перспективи для подальшого розвитку і розширення його можливостей.

**Ключові слова:** прогнозування часових рядів; системний підхід; ймовірнісні, статистичні і параметричні невизначеності; система підтримки прийняття рішень.

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#### ОБРАБОТКА НЕОПРЕДЕЛЕННОСТЕЙ ПРИ МОДЕЛИРОВАНИИ НЕСТАЦИОНАРНЫХ ВРЕМЕННЫХ РЯДОВ С ИСПОЛЬЗОВАНИЕМ СИСТЕМ ПОДДЕРЖКИ РЕШЕНИЙ

**Проблематика.** Прогнозирование нелинейных нестационарных временных рядов (ННВР) – важная задача для экономики, маркетинга, промышленности, экологии и многих других отраслей науки и практической деятельности. Для успешного решения этой задачи необходимо разрабатывать современные компьютерные системы поддержки принятия решений (СППР), которые дадут возможность получать надежные оценки прогнозов в условиях наличия неопределенностей различных типов и природы.

**Цель исследования.** Разработка требований к современным СППР и их формальное представление; анализ типов неопределенностей, характерных для процессов построения математических моделей и прогнозирования; выбор методов учета возможных неопределенностей; иллюстрация применения системы для решения задачи прогнозирования гетероскедастических ННВР с помощью статистических данных.

**Методика реализации.** Для достижения поставленной цели использованы такие методы: системный подход к анализу статистических данных; статистический подход к идентификации и учету возможных неопределенностей; алгоритмы калмановской фильтрации; подход на основе байесовского программирования и множества статистических критериев адекватности моделей и качества прогнозов.

**Результаты исследования.** Предложены формальное описание СППР и требования к их разработке; определены классы математических методов, необходимых для успешной реализации СППР; предложены некоторые подходы к формальному учету вероятностных, статистических и параметрических неопределенностей; приведен иллюстративный пример использования СППР.

**Выводы.** Системный подход к созданию СППР для решения задач прогнозирования нелинейных нестационарных временных рядов позволяет успешно решать поставленную задачу. Используя предложенную систему, можно учитывать неопределенности вероятностного, статистического и параметрического типов и вычислять высококачественные оценки краткосрочных и среднесрочных прогнозов ННВР. Предложенный подход имеет хорошие перспективы для дальнейшего развития и расширения его возможностей.

**Ключевые слова:** прогнозирование временных рядов; системный подход; вероятностные, статистические и параметрические неопределенности; система поддержки принятия решений.

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