

# ЕЛЕКТРОНІКА, РАДІОТЕХНІКА ТА ТЕЛЕКОМУНІКАЦІЇ

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## $TE_{01\delta}$ -, $TE_{02\delta}$ -DUALBAND FILTER ON DIELECTRIC RESONATORS

This paper is concerned with a challenging problem of dual-band filter design based on rectangular dielectric resonators (DR) with no additional coupling elements or complication of DR shape. We exploit the possibility of operating on both higher  $TE_{02\delta}$  oscillation mode ensuring higher values of unloaded Q-factor and the lower one  $TE_{01\delta}$ . Based on analytical scattering models with one and two DRs, calculated coupling coefficients, the analytical model of dual-band filter has been developed, that accounts for two bands simultaneously. The applied methods include the analytical, finite element method and the experimental one. The accuracy of the presented analytical model has been verified by simulation results using finite element method as well as by the conducted experiment, that show very good agreement. The proposed structure can be used in front-ends of multi-band applications in wireless networks after adjustment to the desired frequency range.

**Keywords:** dielectric resonator; oscillation mode; dual-band filter; coupling coefficient.

### Introduction

While constructing the front-ends for satellite communications, there is a strong need in multi-band band-pass filters [1] in order to provide several frequency bands simultaneously. Whereas the existing constructions of multi-band filters on micro-strip resonators and combline structures have the limited usage in the microwave range due to the increased losses in metal, the resonators with higher unloaded Q-factor are of great interest, such as the dielectric resonators (DRs) [2, 3].

There are two main constructions of dual-band filters on DRs. The first one utilizes the union of two rectangular  $TE_{01\delta}$  dielectric rings and has a complicated shape in the form of number "eight" [4]. The second one consists of the half-cut of cylindrical DRs, coupled through either cross-shaped [5] or rectangular irises [6] with two hybrid  $HEE_{11\delta}$ ,  $HEH_{11\delta}$  degenerated operating modes. The drawbacks of the above mentioned structures include: the general complication of the DR's shape; the increased radiation losses, consequently leading to the increased insertion losses in the pass-band due to the operation in two lower  $TE_{01\delta}$  oscillation modes (particularly for the first construction); the necessary usage of additional coupling elements such as irises, tuning screws etc.

The proposed in the current article construction of dual-band filter consists of rectangular shaped DRs for more flexible frequency control in terms of additional degree of freedom in dimensions compared with the cylindrical DRs; the adjacent operating  $TE_{01\delta}$  and  $TE_{02\delta}$  modes; absence of additional coupling elements or shape complication. The opportunity of  $TE_{02\delta}$  higher mode usage

with greater unloaded Q-factor has been described in [7, 8] for DRs with elongated shape along one of the axes.

### Problem statement

Possibility of C-frequency range (4–7 GHz) dual-band filter synthesis with  $TE_{01\delta}$  and  $TE_{02\delta}$  operational modes of rectangular DRs with frequency band up to 2% is under research. The applied methods include the analytical, finite element method (FEM) [9] and the experimental one.

In order to solve the problem it is required to:

- 1) calculate the resonant frequencies of rectangular DR in  $TE_{01\delta}$  and  $TE_{02\delta}$  modes for chosen dimensions of DR;
- 2) by using the obtained analytical scattering models with one and two DRs [8] for chosen dimensions and material of DR, calculate the coupling coefficients of DR with the transmission line as well as the cross-coupling coefficients between the DRs for both oscillation modes;
- 3) calculate the amplitude-frequency response in each band applying the above-mentioned methods.

### Resonant frequency calculation of DR

In articles [7, 8, 10] the mode charts' frequency dependencies of the rectangular DR have been researched for elongated shapes of DR ( $a_0 = b_0 \approx 0.5 \cdot L_0$ , where  $a_0 \cdot L_0 \cdot b_0$  are dimensions of DR in fig. 1). The electro-magnetic field (EMF) distribution for two operating modes, namely  $TE_{01\delta}$  and  $TE_{02\delta}$  inside DR, has been illustrated in fig. 1. Taking into account that the

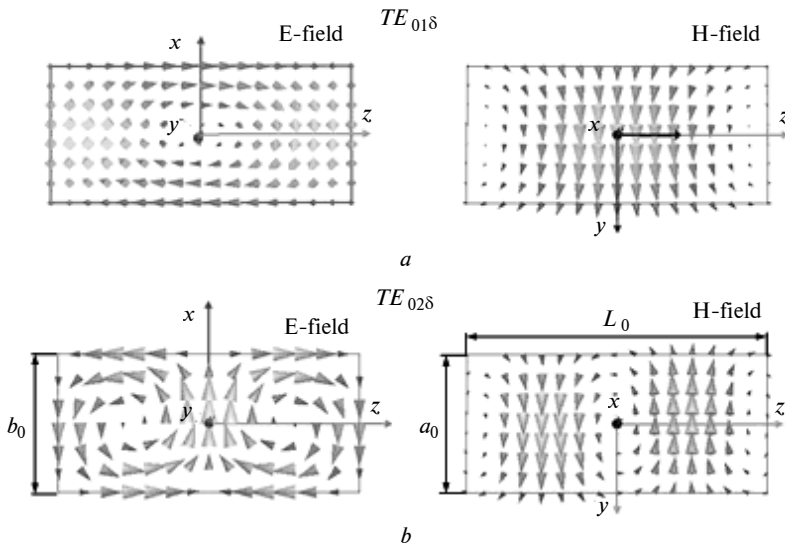


Fig. 1. EMF distribution of  $TE_{01\delta}$  (a) and  $TE_{02\delta}$  (b)

operating  $TE_{02\delta}$  eigen mode has more concentrated EMF distribution inside the DR and resembles two closely located antiphase magnetic dipoles in fig.1, b, it decays more rapidly compared with the basic  $TE_{01\delta}$  mode while withdrawing from the surface of the DR, thus leading to the increase in Q-factor.

Let the optimal dimensions of DR on the criterion of maximal frequency separation from the operating modes have been chosen as  $a_0 = b_0 = 6.4$  mm while  $L_0 = 14$  mm for the given frequency band, where DR has  $\epsilon_r = 38.9$  as well as  $\text{tg}\delta = 5 \cdot 10^{-4}$ . The resonant frequencies of such DR in  $TE_{01\delta}$  and  $TE_{02\delta}$  modes have been calculated by applying three methods, namely: analytical by solving the transcendental equation [7], FEM and experimental one.

The transcendental equation for both modes is:

$$\beta_y \cdot \tan\left(\frac{\beta_y \cdot b_0}{2}\right) = \beta_{0y}, \quad (1)$$

where  $\beta_y = \sqrt{\epsilon_r k_0^2 - (\beta_x^2 + \beta_z^2)}$  – wave number along  $oY$  axis inside DR;  $\beta_{0y} = \sqrt{k_0^2(\epsilon_r - 1) - \beta_y^2}$ ;  $k_0$  – wave number in the vacuum;  $\epsilon_r = 38.9$  – relative electric permittivity of dielectric;  $\beta_z = \frac{2\pi m}{L_0}$ ,

$\beta_x = \frac{\pi}{a_0}(2n+1)$ ;  $m=2, n=0$  – wave numbers of  $TE_{02\delta}$  (higher mode) along  $oZ$  and  $oX$  axes inside

DR correspondingly;  $\beta_z = \frac{\pi}{L_0}(2m+1)$ ,

$\beta_x = \frac{\pi}{a_0}(2n+1)$ ;  $m=1, n=0$  – wave numbers for  $TE_{01\delta}$  (lower mode) along  $oZ$  and  $oX$  inside DR correspondingly.

The resonant frequencies calculated according to the transcendental equation (1), have been summarized in table 1, where  $f_{0_1}$ ,  $f_{0_2}$  – resonant frequencies of  $TE_{01\delta}$  and  $TE_{02\delta}$  modes respectively.

For more exact determination of resonant frequencies and EMF distribution of first four modes the FEM has been applied to the simulation model in fig. 2 [9]. It consists of DR with  $\epsilon_r = 38.9$  and  $\text{tg}\delta = 5 \cdot 10^{-4}$ , located in the metal cavity with dimensions of  $32 \times 32 \times 70$  mm.

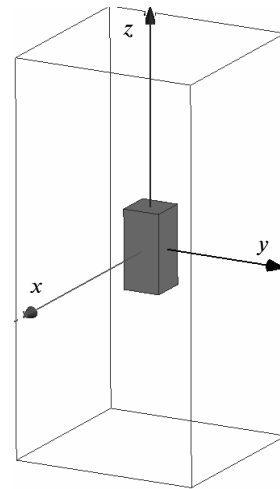
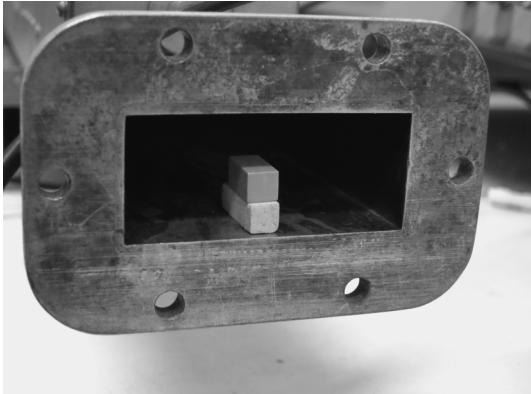
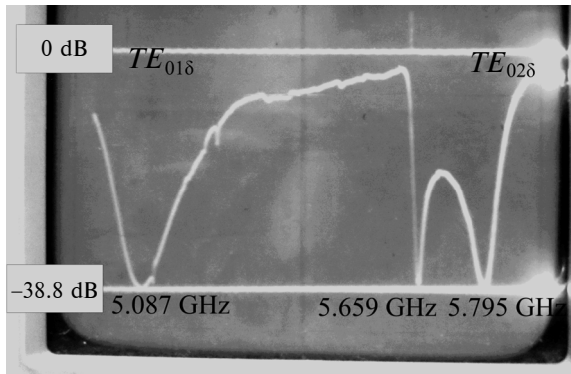


Fig. 2. The simulation model for eigenmode definition

The part of test – bench has been illustrated in fig. 3, a that consists of DR, located on the plastic foam support inside the waveguide directional coupler as well as the experimental oscillogram of the obtained results for operational modes and spurious one. As it can be observed from fig. 3, b the narrowband spurious product is located between the operational  $TE_{01\delta}$  and  $TE_{02\delta}$  modes. Also  $TE_{02\delta}$  is more narrowband compared with  $TE_{01\delta}$ , where the – 3 dB bandwidth of each doesn't exceed 3%. The obtained results of the conducted experiment have been summarized in table 1.



a



b

Fig. 3. The view of DR on the plastic foam support (a) and  $|S_{21}|$  measured data (b)

The results from the application of these methods were compared to each other and summarized in table 1.

**Table 1.** The DR resonant frequency calculation of  $6.4 \times 6.4 \times 14$  mm size with  $\epsilon_r = 38.9$ ,  $\text{tg}\delta = 5 \cdot 10^{-4}$

Parameter	Method of calculation		
	Transcendental equation (1)	FEM [9]	Experiment
$f_{0\_1}$ , GHz	5.21044	5.06896	5.087
$f_{0\_2}$ , GHz	5.7212	5.83806	5.795
inaccuracy $f_{0\_1}$ , %	2.8	0.3	—
inaccuracy $f_{0\_2}$ , %	1.3	0.7	—
$f_{0\_sp}$ , GHz	—	5.605	5.659

Hence, according to the data from table 1, the inaccuracy of less than 1 % regarding the experiment can be obtained by applying FEM [9], with the inaccuracy of less than 1 %, while the inaccuracy of analytical solution is also rather tolerant and doesn't overcome 3 %.

### The coupling coefficients of dual-band filter

In order to calculate the coupling coefficients of DR with the transmission line  $\tilde{k}_{11}$  as well as the cross-coupling coefficients  $k_{12}$  between DRs, firstly, the frequency dependencies of transmission coefficients  $T_1(f)$ ,  $T_2(f)$  of the simulated models with one and two DRs relatively [8] have been evaluated using FEM. Thus, by knowing  $T_1(f)$ ,  $T_2(f)$  on the central frequency of each band the unknown quantities of coupling coefficients can be determined based on the analytical scattering models with one and two DRs [8]:

$$T_1(f) = - \left( \frac{2 \cdot \tilde{k}_{11\_j} Q^D}{\frac{f}{f_{0\_j}} + 2i Q^D \left( \frac{f}{f_{0\_j}} - 1 - \frac{\lambda_j}{2} \right)} \right), \quad (2)$$

$$T_2(f) = - \frac{\tilde{k}_{11\_j} Q^D}{\frac{f}{f_{0\_j}} + 2i Q^D \left( \frac{f}{f_{0\_j}} - 1 - \frac{\lambda_{1\_j}}{2} \right)} +$$

$$\frac{\tilde{k}_{11\_j} Q^D}{\frac{f}{f_{0\_j}} + 2i Q^D \left( \frac{f}{f_{0\_j}} - 1 - \frac{\lambda_{2\_j}}{2} \right)}, \quad (3)$$

where  $T_1(f)$ ,  $T_2(f)$  – the transmission coefficients of the structure with one and two DR correspondingly;  $Q^D$  – Q-factor of dielectric;  $f_{0\_j}$  – the resonance frequency of  $j$ -th operating mode;  $j = 1, 2$  corresponds to  $TE_{018}$  and  $TE_{028}$  modes;  $\lambda_j = 2 \cdot i \cdot \tilde{k}_{11\_j}$ ; here  $\tilde{k}_{11\_j}$  is the unknown coupling coefficient between the transmission line and DR in  $j$ -th operating mode;  $\lambda_{1\_j} = i \tilde{k}_{11\_j} + k_{12\_j}$ ,  $\lambda_{2\_j} = i \tilde{k}_{11\_j} - k_{12\_j}$  eigenvalues of the coupling operator, corresponding to the  $j$ -th eigenmode of DR that determine the frequencies and amplitudes of the coupled modes of the structure with two DRs,  $k_{12\_j}$  is the unknown cross-coupling coefficient between the DRs.

Assuming that the transmission coefficients of the structure for each band are known quantities (by applying the FEM [9]) as well as the coupling coefficients  $\tilde{k}_{11\_j}$  (calculated according to (2)),

the cross-coupling coefficients  $k_{12\_j}$  between the DRs can be found separately for each band ( $j = 1, 2$ ) according to (3).

**Table 2.** Coupling coefficients of DR

Parameters	1 <sup>st</sup> band	2 <sup>nd</sup> band
$\tilde{k}_{11\_1}, \tilde{k}_{11\_2}$	3.822e-3	2.769e-3
$k_{12\_1}, k_{12\_2}$	6.38579e-3	2.57556e-3

The acceptable for filter synthesis numerical values of coupling coefficients for both modes have been summarized in table 2 for the case  $dz = 5.8$  mm,  $dp = 8.45$  mm (fig. 4, a). Previously, the basic  $TE_{018}$  and adjacent higher  $TE_{028}$  mode of elongated shapes of rectangular DRs ( $a_0 = b_0 \approx 0.5 \cdot L_0$ ) were deduced to have approximately the same dependencies of coupling coefficients with the variation of the structure parameters, thus enabling to develop dual-band filters. The calculated value of  $k_{12\_1}$  coupling between

DRs is greater than critical, that can be explained by difficulties of precise location of two DRs inside the metal cavity while conducting the experiment.

### Amplitude frequency response of dual-band filter

By providing the simultaneous decomposition of the scattering field in two coupled modes  $TE_{018}$ ,  $TE_{028}$  the analytical expressions for transmission coefficient  $T_3(f)$  as well as reflection coefficient of the filter  $R_3(f)$  have been obtained, that take into account two bands simultaneously:

$$T_3(f) = -\sum_{j=1}^2 \frac{\tilde{k}_{11\_j} Q^D}{\frac{f}{f_{0\_j}} + 2i Q^D \left( \frac{f}{f_{0\_j}} - 1 - \frac{\lambda_{1\_j}}{2} \right)} + \sum_{j=1}^2 \frac{\tilde{k}_{11\_j} Q^D}{\frac{f}{f_{0\_j}} + 2i Q^D \left( \frac{f}{f_{0\_j}} - 1 - \frac{\lambda_{2\_j}}{2} \right)}, \quad (4)$$

$$R_3(f) =$$

$$1 - \sum_{j=1}^2 \left( \frac{\tilde{k}_{11\_j} Q^D}{\frac{f}{f_{0\_j}} + 2i Q^D \left( \frac{f}{f_{0\_j}} - 1 - \frac{\lambda_{1\_j}}{2} \right)} \right) - \sum_{j=1}^2 \left( \frac{\tilde{k}_{11\_j} Q^D}{\frac{f}{f_{0\_j}} + 2i Q^D \left( \frac{f}{f_{0\_j}} - 1 - \frac{\lambda_{2\_j}}{2} \right)} \right),$$

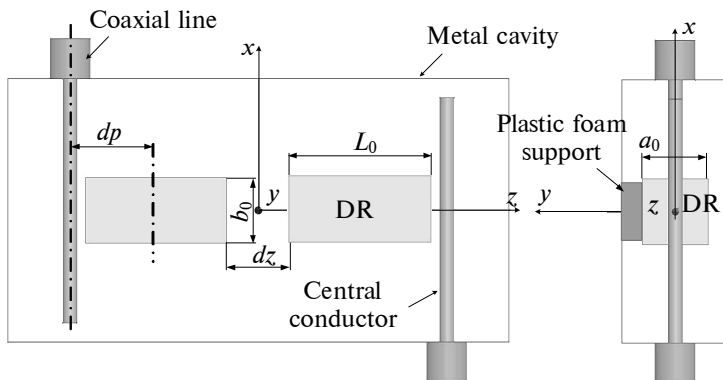
where  $j = 2$  corresponds to the number of pass-bands;

$$\lambda_{1\_j} = i\tilde{k}_{11\_j} + k_{12\_j},$$

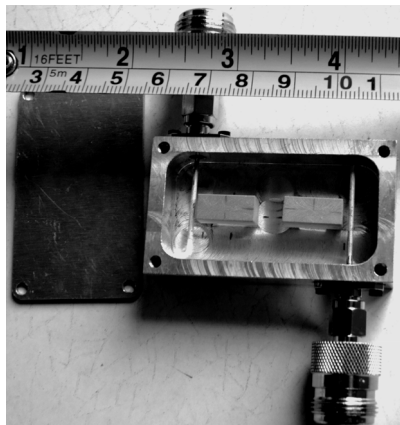
$$\lambda_{2\_j} = i\tilde{k}_{11\_j} - k_{12\_j}.$$

In fig. 4 the construction of the examined dual-band filter and the photo of trial model have been shown.

The construction consists of metal cavity of  $10.6 \times 50 \times 26$  mm size, two DRs  $6.4 \times 6.4 \times 14$  mm, fabricated from material with  $\epsilon_r = 38.9$  and  $\text{tg}\delta = 5 \cdot 10^{-4}$ , and metal pins, connected to the coaxial lines.



a



b

**Fig. 4.** Construction of the investigated dual-band filter on rectangular DR (a) and the view of trial model (b)

According to the chosen coupling coefficients (table 2) the experimental amplitude-frequency response (AFR) of dual-band filter has been compared with the AFR based on analytical model (4) as well as the results of simulation by applying FEM [9] in fig. 5 (a, b). The phase frequency responses (PFR) of  $|S_{21}|$  in each band have been also illustrated in fig. 5 (c, d).

According to the data from fig. 7 the resulting AFR and PFR have little discrepancies while using three methods thus showing good consistency of results.

The obtained quality indices of the proposed dual-band filter have been summarized in table 3 according to three different calculation approaches.

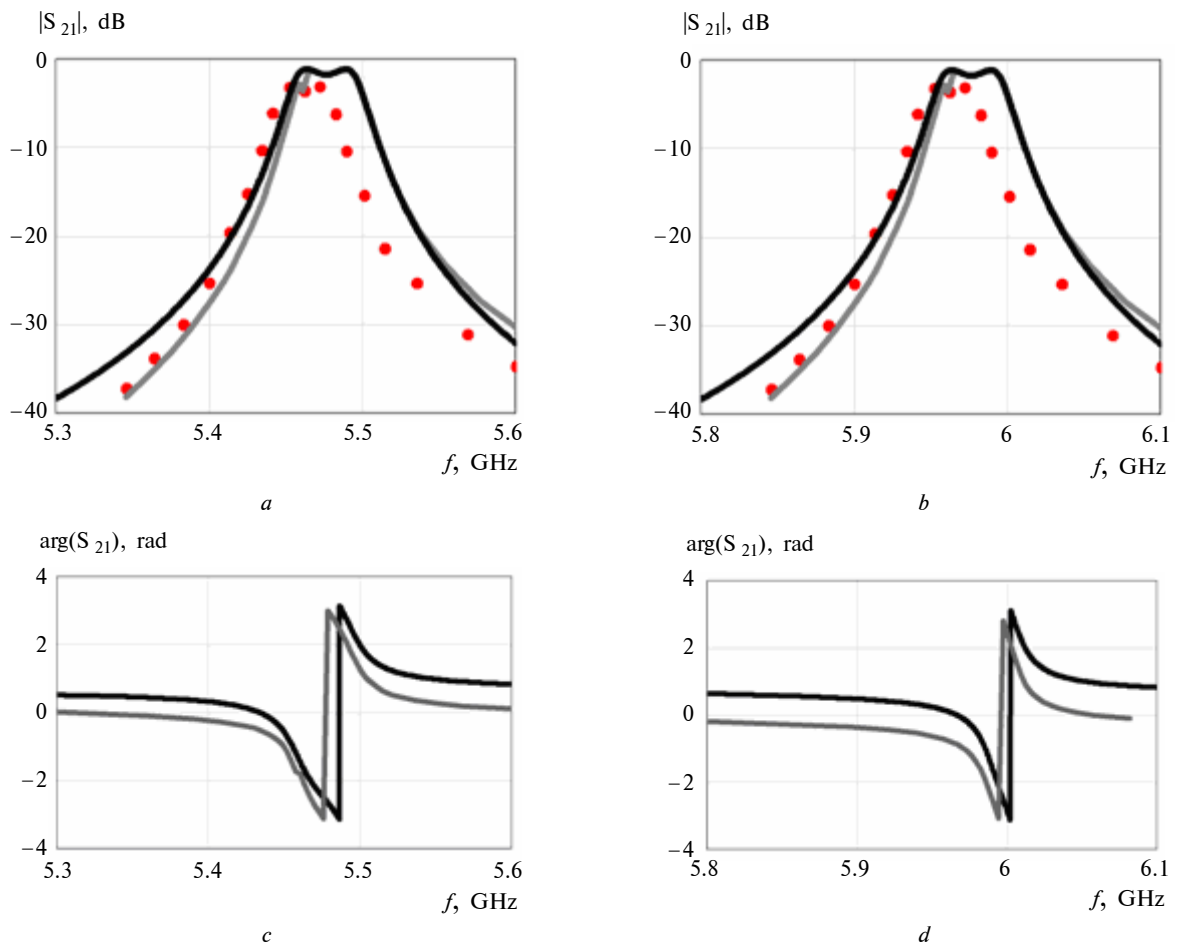


Fig. 5. The comparison of the calculated  $|S_{21}|$  and phase in the first (a, c) and in the second bands (b, d): ●●● – experiment, — — analytical model (4); — — FEM

**Table 3.** Comparison of quality indices of the dual-band dielectric resonator filter by applying analytical method (3), FEM and the experimental one

Parameter	First band/ Second band (4)	First band/ Second band FEM	First band/ Second band experiment
Central frequency of the pass-band, GHz	5.476/5.997	5.48/5.996	5.462/5.992
3db Bandwidth, MHz (%)	48/22 MHz 0.88/0.37 %	40/22 MHz 0.73/0.37 %	41/25 MHz 0.75/0.42 %
Minimal inband losses, dB	1.07/1.7	1.12/1.68 dB	3.11/4.43 dB
Central frequency of the adjacent spurious product, GHz	—	6.271	6.268
Frequency attenuation between bands, dB	43	39.9	40

From table 3 the results based on FEM and analytical model illustrate very good consistency together with the experimental one. Consequently, the new construction of dual-band filter has been proposed, based on DR of rectangular shape with a square cross-section, the height of which is approximately twice of the base. The distinctive feature of the proposed construction is operation on two modes, namely  $TE_{01\delta}$ ,  $TE_{02\delta}$ ; furthermore, absence of the additional coupling elements or complication of the DR's shape, as well as an opportunity to use it in microwave range.

### Conclusions

The new construction of dual-band filter on the elongated rectangular DRs has been developed,

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