ІНФОРМАЦІЙНІ ТЕХНОЛОГІЇ, СИСТЕМНИЙ АНАЛІЗ ТА КЕРУВАННЯ

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LOGICAL OPERATIONS IN ALGEBRAIC SYSTEM OF AGGREGATES FOR MULTIMODAL DATA REPRESENTATION AND PROCESSING

Background. The range of computer software applications which operate with multimodal data becomes wider and wider. To develop efficient algorithms for data processing, the representation of multimodal data as complex structures is required. Existing approaches are mostly oriented to independent data sets representation and they are not efficient for complex representation of multimodal data.
Objective. The development of a mathematical approach which can be used for complex multimodal data representation and processing in computer systems.
Methods. The Algebraic System of Aggregates is developed for enabling complex representation of multimodal data. There are logical, ordering and arithmetical operations in the Algebraic System of Aggregates. Logical operations can be used for preparing complex data representations in a form of aggregates which are supposed to be a subject of further data analysis. The precondition for such representation is that data sequences are of different modalities and they are recorded with respect to time.
Results. The logical operations on aggregates are proposed and described. They allow to construct different compositions of multimodal data what in its turn enables complex data representation for compound description of objects and processes in different areas including healthcare. The multi-image concept which enables overall description of an object, a subject, or a process of observation carried out in the course of time is also proposed and discussed.
Conclusions. The Algebraic System of Aggregates is a tool for complex data representation. It enables multimodal data processing by using a range of operations and relations. The key difference in the fulfilment of logical operations in the Algebraic System of Aggregates and logical operations on sets is that an aggregate is a form of complex representation of data sequences where the order of data in each sequence of each type is important and influences on the order of data in the result of a certain logical operation.
Keywords: algebraic system of aggregates; multi-image; multimodal data; logical operations.

Introduction

Nowadays, the range of computer software applications which operate with multimodal data becomes wider and wider. To develop efficient algorithms for data processing, we need to represent multimodal data as complex structures. From mathematical perspective, such representation can be supported to some extent by tuples algebra and multisets theory [1—10]. *Tuples Algebra* (TA) is a mathematical system which enables modelling the theory of polyadic relations [1—4]. The TA is an algebraic system, carrier of which is an arbitrary set of polyadic relations expressed in specific structures, such as C-tuple, C-system, D-tuple, D-system, which are called TA-objects [2, 4].

Let us assume that there is a family of different sets named *kinds*. Then every kind is associated with a certain set of *attributes*. A domain of each attribute is a set related to a certain kind. Attribute domains correspond to variables domain in mathematical logics and they correspond to feature scales in information systems.

A *flexible universe* consists of a certain family of *partial universes* which are Cartesian products of domains for a given sequence of attributes. The sequence of attribute names which define the given partial universe is named a *relation schema*. TA-objects formed in the same relation schema are named *homotypic*.

Thus, the TA is an interpretation of mathematical logic. It allows to use it for modelling and analysis of logical reasoning and, therefore, for knowledge representation. It makes the TA attractive for using in multimodal data representation. However, in many cases, we face a necessity to represent multimodal data not as a set of independent data but as a complex object described by these data sequences. Thus, we need to use another approach to meet the requirement of multimodal data sequences complex representation.

Problem Statement

The objective of the research is to develop a mathematical approach which can be used for

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complex multimodal data representation and processing in computer systems.

Algebraic System of Aggregates

An *Algebraic System* (algebraic structure) is an object which consists of sets ($\mathcal{M}$, $\mathcal{F}$, $\mathcal{R}$) which satisfy some system of axioms, where $\mathcal{M}$ is a non-empty set (carrier), elements of which are elements of the system; $\mathcal{F}$ is a set of operations; $\mathcal{R}$ is a set of relations. An algebraic system with an empty set of relations is called an *algebra* and a system with an empty set of operations is called a *model* [11].

A *tuple* is an ordered set of elements. $X^{<\omega} = \bigcup_{n\in\mathrm{N}} X^{n}$ is a set of all tuples of arbitrary length which consists of elements of a set *X*. In particular, $2^{<\omega}$ is a set of all dyadic tuples. [3]

The *Algebraic System of Aggregates* (ASA) is an algebraic system, a carrier of which is an arbitrary set of specific structures – *aggregates* [12].

An aggregate *A* is an ordered set of tuples of arbitrary lengths *l*, *m*, *n*, ..., *q*:

$$\langle a_1, a_2, \ldots, a_l\rangle, \langle b_1, b_2, \ldots, b_m\rangle, \langle c_1, c_2, \ldots, c_n\rangle, \ldots,$$
$$\langle w_1, w_2, \ldots, w_q\rangle,$$

elements of which belong to a certain set: $a_i \in M_1, b_j \in M_2, c_k \in M_3, \ldots, w_u \in M_N$. Thus, the aggregate is a tuple of arbitrary tuples.

Since the order of sets which are used for composing of tuples in the aggregate is important for carrying out operations on aggregates, let us use the following notation of the aggregate:

$$A = [\![M_1, M_2, M_3, \ldots, M_N \mid \langle a_1, a_2, \ldots, a_l\rangle,$$
$$\langle b_1, b_2, \ldots, b_m\rangle, \langle c_1, c_2, \ldots, c_n\rangle, \ldots \langle w_1, w_2, \ldots, w_q\rangle]\!]$$
$$= [\![M_1, M_2, M_3, \ldots, M_N \mid \bar{a}, \bar{b}, \bar{c}, \ldots, \bar{w}]\!].$$

The meaning of this notion is that at first we indicate the sets, elements of which belong to the aggregate within appropriate tuples, and then we place the tuples of the elements belonging to these sets in the same order because there is a strict relation between them: the elements of the first tuple belong to the first set, the elements of the second tuple belong to the second set, etc. The sequence order of sets and corresponding tuples defines how operations on aggregates will be fulfilled.

The sets in the list of sets may repeat. Such case means that the aggregate includes several tuples which consist of elements of the same type.

Tuple elements can be both strict and fuzzy values. The notation a_i means that a value is strict and the notation $\tilde{a}_i$ is used for a fuzzy value. A tuple element can be undefined; its notation is _ or $\tilde{\_}$ depending on the value type.

A tuple may be empty. Its notation is $\langle\varnothing\rangle$:

$$A = [\![M_1, M_2, M_3, \ldots, M_N \mid \langle a_1, a_2, \ldots, a_l\rangle,$$
$$\langle\varnothing\rangle, \langle c_1, c_2, \ldots, c_n\rangle, \ldots, \langle w_1, w_2, \ldots, w_q\rangle]\!].$$

An aggregate which consists only of empty tuples is called *empty*:

$$A = [\![M_1, M_2, M_3, \ldots, M_N \mid \langle\varnothing\rangle, \langle\varnothing\rangle, \langle\varnothing\rangle, \ldots, \langle\varnothing\rangle]\!]$$
$$= [\![M_1, M_2, M_3, \ldots, M_N \mid \langle\varnothing\rangle]\!].$$

An aggregate which does not include any component is called a *null-aggregate*, its notation is $A_\varnothing$:

$$A_\varnothing = [\![\varnothing \mid \langle\varnothing\rangle]\!].$$

The null-aggregate plays a role of a neutral element in the algebraic system of aggregates.

An *undefined aggregate* is the aggregate which includes tuples of defined sets but unknown elements values. It defers from the empty aggregate where tuples also belong to defined sets, but the tuples are empty. The practical value of the notion of an undefined aggregate is that it allows us to predefine aggregates when some data sequences are not obtained yet (e.g. we know from which sensor we will receive data, but the sensor is currently off and, thus, data are still unavailable).

The notation of the undefined aggregate is:

$$A = [\![M_1, M_2, M_3, \ldots, M_N \mid \langle\_\rangle, \langle\_\rangle, \langle\_\rangle, \ldots, \langle\_\rangle]\!]$$
$$= [\![M_1, M_2, M_3, \ldots, M_N \mid \langle\_\rangle]\!].$$

The aggregate *length* is a quantity of tuples in it. The corresponding notation is $|A|$. The length of the null-aggregate is equal to zero: $|A_\varnothing| = 0$.

The tuple *length* is a quantity of elements in it. The notation is $|\bar{a}|$.

A *cumulative length* $\|A\|$ of the aggregate *A* is a sum of the lengths of the aggregate tuples. The cumulative length of the null-aggregate is equal to zero: $\|A_\varnothing\| = 0$.

Let us consider two aggregates A_1 and A_2 as *compatible* if they have equal lengths and both the type and sequence order of these aggregates are the same. We use notation $A_1 \doteqdot A_2$ for compatible aggregates.

Let us consider two aggregates A_1 and A_2 as *quasi-compatible* if the type and sequence order of these aggregates coincide partly. There is no requirement of the equality of aggregates lengths in this case. We use notation $A_1 \doteq A_2$ for quasi-compatible aggregates.

Otherwise, let us consider the aggregates A_1 and A_2 as *incompatible*. We use notation $A_1 \mathring{=} A_2$ for incompatible aggregates.

The *logical operations* on aggregates are:

1. Union $\cup$.
2. Intersection $\cap$.
3. Difference \.
4. Symmetric Difference Δ.
5. Exclusive Intersection $\neg$.

Let us consider these operations.

A *union* of two aggregates A_1 and A_2 is the aggregate A_3 which includes components of both aggregates and is formed according to the following rule:

1. If $A_1 \doteqdot A_2$ then elements of *i*-tuple of A_2 are added into the end of *i*-tuple of A_1:

$$A_1 = [\![M_1, M_2, \ldots, M_N \mid \langle a_1^1, a_2^1, \ldots, a_l^1 \rangle, \langle b_1^1, b_2^1, \ldots, b_m^1 \rangle, \ldots, \langle w_1^1, w_2^1, \ldots, w_n^1 \rangle]\!],$$

$$A_2 = [\![M_1, M_2, \ldots, M_N \mid \langle a_1^2, a_2^2, \ldots, a_r^2 \rangle, \langle b_1^2, b_2^2, \ldots, b_q^2 \rangle, \ldots, \langle w_1^2, w_2^2, \ldots, w_p^2 \rangle]\!],$$

$$A_3 = A_1 \cup A_2 = [\![M_1, M_2, \ldots, M_N \mid \langle a_1^1, a_2^1, \ldots, a_l^1, a_1^2, a_2^2, \ldots, a_r^2 \rangle, \langle b_1^1, b_2^1, \ldots, b_m^1, b_1^2, b_2^2, \ldots, b_q^2 \rangle, \ldots, \langle w_1^1, w_2^1, \ldots, w_n^1, w_1^2, w_2^2, \ldots, w_p^2 \rangle]\!]. \quad (1)$$

2. If $A_1 \mathring{=} A_2$ then the tuples of A_2 are added into the end of the tuple of tuples of A_1 and the corresponding sets of A_2 are added into the end of the sets sequence of A_1:

$$A_1 = [\![M_1^1, M_2^1, \ldots, M_N^1 \mid \langle a_1, a_2, \ldots, a_l \rangle, \langle b_1, b_2, \ldots, b_m \rangle, \ldots, \langle w_1, w_2, \ldots, w_n \rangle]\!],$$

$$A_2 = [\![M_1^2, M_2^2, \ldots, M_K^2 \mid \langle c_1, c_2, \ldots, c_r \rangle, \langle d_1, d_2, \ldots, d_q \rangle, \ldots, \langle z_1, z_2, \ldots, z_p \rangle]\!],$$

$$A_3 = A_1 \cup A_2 = [\![M_1^1, M_2^1, \ldots, M_N^1, M_1^2, M_2^2, \ldots, M_K^2 \mid \langle a_1, a_2, \ldots, a_l \rangle, \langle b_1, b_2, \ldots, b_m \rangle, \ldots, \langle w_1, w_2, \ldots, w_n \rangle, \langle c_1, c_2, \ldots, c_r \rangle, \langle d_1, d_2, \ldots, d_q \rangle, \ldots, \langle z_1, z_2, \ldots, z_p \rangle]\!]. \quad (2)$$

3. If $A_1 \doteq A_2$ then elements of *i*-tuple of A_2 are added into the end of *i*-tuple of A_1 if elements of these tuples belong to the same set; otherwise, the rule for incompatible aggregates is applied:

$$A_1 = [\![M_1, M_2^1, \ldots, M_x, \ldots, M_N^1 \mid \langle a_1^1, a_2^1, \ldots, a_l^1 \rangle, \langle b_1, b_2, \ldots, b_m \rangle, \ldots, \langle f_1^1, f_2^1, \ldots, f_t^1 \rangle, \ldots, \langle w_1, w_2, \ldots, w_n \rangle]\!],$$

$$A_2 = [\![M_1, M_2^2, \ldots, M_x, \ldots, M_K^2 \mid \langle a_1^2, a_2^2, \ldots, a_r^2 \rangle, \langle d_1, d_2, \ldots, d_q \rangle, \ldots, \langle f_1^2, f_2^2, \ldots, f_v^2 \rangle, \ldots, \langle z_1, z_2, \ldots, z_p \rangle]\!],$$

$$A_3 = A_1 \cup A_2 = [\![M_1, M_2^1, \ldots, M_x, \ldots, M_N^1, M_2^2, \ldots, M_K^2 \mid \langle a_1^1, a_2^1, \ldots, a_l^1, a_1^2, a_2^2, \ldots, a_r^2 \rangle \langle b_1, b_2, \ldots, b_m \rangle, \ldots, \langle f_1^1, f_2^1, \ldots, f_t^1, f_1^2, f_2^2, \ldots, f_v^2 \rangle, \ldots, \langle w_1, w_2, \ldots, w_n \rangle, \langle d_1, d_2, \ldots, d_q \rangle, \ldots, \langle z_1, z_2, \ldots, z_p \rangle]\!]. \quad (3)$$

An *intersection* of two aggregates A_1 and A_2 is the aggregate A_3 which includes only common components of both aggregates and is formed according to the following rule:

1. If $A_1 \doteqdot A_2$ then A_3 includes elements of both aggregates, which are common for them, in every tuple:

$$A_1 = [\![M_1, M_2, \ldots, M_N \mid \langle a_1^1, a_2^1, \ldots, a_l^1 \rangle, \langle b_1^1, b_2^1, \ldots, b_m^1 \rangle, \ldots, \langle w_1^1, w_2^1, \ldots, w_n^1 \rangle]\!],$$

$$A_2 = [\![M_1, M_2, \ldots, M_N \mid \langle a_1^2, a_2^2, \ldots, a_r^2 \rangle, \langle b_1^2, b_2^2, \ldots, b_q^2 \rangle, \ldots, \langle w_1^2, w_2^2, \ldots, w_p^2 \rangle]\!],$$

$$A_3 = A_1 \cap A_2 = [\![M_1, M_2, \ldots, M_N \mid \langle a_{l_1}^1, \ldots, a_{l_\alpha}^1, a_{r_1}^2, \ldots, a_{r_\beta}^2 \rangle, \langle b_{m_1}^1, \ldots, b_{m_\gamma}^1, b_{q_1}^2, \ldots, b_{q_\delta}^2 \rangle, \ldots, \langle w_{n_1}^1, \ldots, w_{n_\lambda}^1, w_{p_1}^2, \ldots, w_{p_\mu}^2 \rangle]\!], \quad (4)$$

where $a_{l_i}^1 \in \overline{a^1}, a_{l_i}^1 \in \overline{a^2}, i \in \langle 1,\dots,\alpha\rangle; a_{r_j}^2 \in \overline{a^1}$, $a_{r_j}^2 \in \overline{a^2}, j \in \langle 1,\dots,\beta\rangle$; $b_{m_k}^1 \in \overline{b^1}, b_{m_k}^1 \in \overline{b^2}, k \in \langle 1,\dots,\gamma\rangle$; $b_{q_s}^2 \in \overline{b^1}$, $b_{q_s}^2 \in \overline{b^2}, s \in \langle 1,\dots,\delta\rangle$; $w_{n_u}^1 \in \overline{w^1}, w_{n_u}^1 \in \overline{w^2}$, $u \in \langle 1,\dots,\lambda\rangle$; $w_{p_y}^2 \in \overline{w^1}, w_{p_y}^2 \in \overline{w^2}, y \in \langle 1,\dots,\mu\rangle$.

2. If $A_1 \overset{\circ}{=} A_2$ then A_3 is a null-aggregate:

$$A_1 = [\![M_1^1, M_2^1,\dots,M_N^1 \,|\, \langle a_1, a_2,\dots,a_l\rangle, \langle b_1, b_2,\dots,b_m\rangle,\dots,\langle w_1, w_2,\dots,w_n\rangle]\!],$$

$$A_2 = [\![M_1^2, M_2^2,\dots,M_K^2 \,|\, \langle c_1, c_2,\dots,c_r\rangle, \langle d_1, d_2,\dots,d_q\rangle,\dots,\langle z_1, z_2,\dots,z_p\rangle]\!],$$

$$A_3 = A_1 \cap A_2 = [\![\varnothing \,|\, \langle\varnothing\rangle]\!] = A_\varnothing. \quad (5)$$

3. If $A_1 \doteq A_2$ then A_3 includes elements of both aggregates, which are common for them, only in tuples of common sets, thus, the number of sets shortens:

$$A_1 = [\![M_1, M_2^1,\dots,M_x,\dots,M_N^1 \,|\, \langle a_1^1, a_2^1,\dots,a_l^1\rangle, \langle b_1, b_2,\dots,b_m\rangle,\dots,\langle f_1^1, f_2^1,\dots,f_t^1\rangle,\dots,\langle w_1, w_2,\dots,w_n\rangle]\!],$$

$$A_2 = [\![M_1, M_2^2,\dots,M_x,\dots,M_K^2 \,|\, \langle a_1^2, a_2^2,\dots,a_r^2\rangle, \langle d_1, d_2,\dots,d_q\rangle,\dots,\langle f_1^2, f_2^2,\dots,f_v^2\rangle,\dots,\langle z_1, z_2,\dots,z_p\rangle]\!],$$

$$A_3 = A_1 \cap A_2 = [\![M_1,\dots,M_x \,|\, \langle a_{l_1}^1,\dots,a_{l_\alpha}^1, a_{r_1}^2,\dots,a_{r_\beta}^2\rangle,\dots, \langle f_{t_1}^1,\dots,f_{t_\rho}^1, f_{v_1}^2,\dots,f_{v_\omega}^2\rangle]\!], \quad (6)$$

where $a_{l_i}^1 \in \overline{a^1}, a_{l_i}^1 \in \overline{a^2}, i \in \langle 1,\dots,\alpha\rangle; a_{r_j}^2 \in \overline{a^1}$, $a_{r_j}^2 \in \overline{a^2}$, $j \in \langle 1,\dots,\beta\rangle$; $f_{t_e}^1 \in \overline{f^1}, f_{t_e}^1 \in \overline{f^2}, e \in \langle 1,\dots,\rho\rangle$; $f_{v_h}^2 \in \overline{f^1}$, $f_{v_h}^2 \in \overline{f^2}, h \in \langle 1,\dots,\omega\rangle$.

A *difference* of two aggregates A_1 and A_2 is the aggregate A_3 which includes only components present in A_1 and absent in A_2; it is formed according to the following rule:

1. If $A_1 \doteqdot A_2$ then A_3 includes elements of A_1, which are absent in A_2, in every tuple:

$$A_1 = [\![M_1, M_2,\dots,M_N \,|\, \langle a_1^1, a_2^1,\dots,a_l^1\rangle, \langle b_1^1, b_2^1,\dots,b_m^1\rangle,\dots,\langle w_1^1, w_2^1,\dots,w_n^1\rangle]\!],$$

$$A_2 = [\![M_1, M_2,\dots,M_N \,|\, \langle a_1^2, a_2^2,\dots,a_r^2\rangle, \langle b_1^2, b_2^2,\dots,b_q^2\rangle,\dots,\langle w_1^2, w_2^2,\dots,w_p^2\rangle]\!],$$

$$A_3 = A_1 \setminus A_2 = [\![M_1, M_2,\dots,M_N \,|\, \langle a_{l_1}^1,\dots,a_{l_\alpha}^1\rangle, \langle b_{m_1}^1,\dots,b_{m_\gamma}^1\rangle,\dots,\langle w_{n_1}^1,\dots,w_{n_\lambda}^1\rangle]\!], \quad (7)$$

where $a_{l_i}^1 \in \overline{a^1}, a_{l_i}^1 \notin \overline{a^2}, i \in \langle 1,\dots,\alpha\rangle; b_{m_k}^1 \in \overline{b^1}, b_{m_k}^1 \notin \overline{b^2}$, $k \in \langle 1,\dots,\gamma\rangle; w_{n_u}^1 \in \overline{w^1}, w_{n_u}^1 \notin \overline{w^2}, u \in \langle 1,\dots,\lambda\rangle$.

2. If $A_1 \overset{\circ}{=} A_2$ then A_3 is equal to A_1:

$$A_1 = [\![M_1^1, M_2^1,\dots,M_N^1 \,|\, \langle a_1, a_2,\dots,a_l\rangle, \langle b_1, b_2,\dots,b_m\rangle,\dots,\langle w_1, w_2,\dots,w_n\rangle]\!],$$

$$A_2 = [\![M_1^2, M_2^2,\dots,M_K^2 \,|\, \langle c_1, c_2,\dots,c_r\rangle, \langle d_1, d_2,\dots,d_q\rangle,\dots,\langle z_1, z_2,\dots,z_p\rangle]\!],$$

$$A_3 = A_1 \setminus A_2 = A_1 = [\![M_1^1, M_2^1,\dots,M_N^1 \,|\, \langle a_1, a_2,\dots,a_l\rangle, \langle b_1, b_2,\dots,b_m\rangle,\dots, \langle w_1, w_2,\dots,w_n\rangle]\!]. \quad (8)$$

3. If $A_1 \doteq A_2$ then A_3 includes elements of A_1, which are absent in A_2, in tuples of common sets and all tuples of sets defined only in A_1:

$$A_1 = [\![M_1, M_2^1,\dots,M_x,\dots,M_N^1 \,|\, \langle a_1^1, a_2^1,\dots,a_l^1\rangle, \langle b_1, b_2,\dots,b_m\rangle,\dots,\langle f_1^1, f_2^1,\dots,f_t^1\rangle,\dots,\langle w_1, w_2,\dots,w_n\rangle]\!],$$

$$A_2 = [\![M_1, M_2^2,\dots,M_x,\dots,M_K^2 \,|\, \langle a_1^2, a_2^2,\dots,a_r^2\rangle, \langle d_1, d_2,\dots,d_q\rangle,\dots,\langle f_1^2, f_2^2,\dots,f_v^2\rangle,\dots,\langle z_1, z_2,\dots,z_p\rangle]\!],$$

$$A_3 = A_1 \setminus A_2 = [\![M_1, M_2^1,\dots,M_x,\dots, M_N^1 \,|\, \langle a_{i_1}^1,\dots,a_{i_\alpha}^1\rangle, \langle b_1, b_2,\dots,b_m\rangle,\dots,\langle f_{t_1}^1,\dots,f_{t_\rho}^1\rangle,\dots, \langle w_1, w_2,\dots,w_n\rangle]\!], \quad (9)$$

where $a_{l_i}^1 \in \overline{a^1}, a_{l_i}^1 \notin \overline{a^2}, i \in \langle 1,\dots,\alpha\rangle; f_{t_e}^1 \in \overline{f^1}, f_{t_e}^1 \notin \overline{f^2}$, $e \in \langle 1,\dots,\rho\rangle$.

A *symmetric difference* of two aggregates A_1 and A_2 is the aggregate A_3 which includes both components present in A_1 and absent in A_2 and components present in A_2 and absent in A_1; it is formed according to the following rule:

1. If $A_1 \doteqdot A_2$ then A_3 includes elements of A_1, which are absent in A_2, and elements of A_2, which are absent in A_1, in every tuple:

$$A_1 = [\![M_1, M_2, \ldots, M_N \,|\, \langle a_1^1, a_2^1, \ldots, a_l^1 \rangle, \langle b_1^1, b_2^1, \ldots, b_m^1 \rangle, \ldots, \langle w_1^1, w_2^1, \ldots, w_n^1 \rangle]\!],$$

$$A_2 = [\![M_1, M_2, \ldots, M_N \,|\, \langle a_1^2, a_2^2, \ldots, a_r^2 \rangle, \langle b_1^2, b_2^2, \ldots, b_q^2 \rangle, \ldots, \langle w_1^2, w_2^2, \ldots, w_p^2 \rangle]\!],$$

$$A_3 = A_1 \Delta A_2 = [\![M_1, M_2, \ldots, M_N \,|\, \langle a_{l_1}^1, \ldots, a_{l_\alpha}^1, a_{r_1}^2, \ldots, a_{r_\beta}^2 \rangle, \langle b_{m_1}^1, \ldots, b_{m_\gamma}^1, b_{q_1}^2, \ldots, b_{q_\delta}^2 \rangle, \ldots, \langle w_{n_1}^1, \ldots, w_{n_\lambda}^1, w_{p_1}^2, \ldots, w_{p_\mu}^2 \rangle]\!], \quad (10)$$

where $a_{l_i}^1 \in \overline{a^1}, a_{l_i}^1 \notin \overline{a^2}, i \in \langle 1, \ldots, \alpha \rangle; a_{r_j}^2 \notin \overline{a^1}, a_{r_j}^2 \in \overline{a^2}, j \in \langle 1, \ldots, \beta \rangle$; $b_{m_k}^1 \in \overline{b^1}, b_{m_k}^1 \notin \overline{b^2}, k \in \langle 1, \ldots, \gamma \rangle; b_{q_s}^2 \notin \overline{b^1}, b_{q_s}^2 \in \overline{b^2}, s \in \langle 1, \ldots, \delta \rangle$; $w_{n_u}^1 \in \overline{w^1}, w_{n_u}^1 \notin \overline{w^2}, u \in \langle 1, \ldots, \lambda \rangle$; $w_{p_y}^2 \notin \overline{w^1}, w_{p_y}^2 \in \overline{w^2}, y \in \langle 1, \ldots, \mu \rangle$.

2. If $A_1 \mathring{=} A_2$ then A_3 is equal to the union of A_1 and A_2:

$$A_1 = [\![M_1^1, M_2^1, \ldots, M_N^1 \,|\, \langle a_1, a_2, \ldots, a_l \rangle, \langle b_1, b_2, \ldots, b_m \rangle, \ldots, \langle w_1, w_2, \ldots, w_n \rangle]\!],$$

$$A_2 = [\![M_1^2, M_2^2, \ldots, M_K^2 \,|\, \langle c_1, c_2, \ldots, c_r \rangle, \langle d_1, d_2, \ldots, d_q \rangle, \ldots, \langle z_1, z_2, \ldots, z_p \rangle]\!],$$

$$A_3 = A_1 \Delta A_2 = [\![M_1^1, M_2^1, \ldots, M_N^1, M_1^2, M_2^2, \ldots, M_K^2 \,|\, \langle a_1, a_2, \ldots, a_l \rangle, \langle b_1, b_2, \ldots, b_m \rangle, \ldots, \langle w_1, w_2, \ldots, w_n \rangle, \langle c_1, c_2, \ldots, c_r \rangle, \langle d_1, d_2, \ldots, d_q \rangle, \ldots, \langle z_1, z_2, \ldots, z_p \rangle]\!] = A_1 \cup A_2. \quad (11)$$

3. If $A_1 \doteq A_2$ then A_3 includes elements of A_1, which are absent in A_2, and elements of A_2, which are absent in A_1, in tuples of common sets, all tuples of sets defined only in A_1, and all tuples of sets defined only in A_2:

$$A_1 = [\![M_1, M_2^1, \ldots, M_x, \ldots, M_N^1 \,|\, \langle a_1^1, a_2^1, \ldots, a_l^1 \rangle, \langle b_1, b_2, \ldots, b_m \rangle, \ldots, \langle f_1^1, f_2^1, \ldots, f_t^1 \rangle, \ldots, \langle w_1, w_2, \ldots, w_n \rangle]\!],$$

$$A_2 = [\![M_1, M_2^2, \ldots, M_x, \ldots, M_K^2 \,|\, \langle a_1^2, a_2^2, \ldots, a_r^2 \rangle, \langle d_1, d_2, \ldots, d_q \rangle, \ldots, \langle f_1^2, f_2^2, \ldots, f_v^2 \rangle, \ldots, \langle z_1, z_2, \ldots, z_p \rangle]\!],$$

$$A_3 = A_1 \Delta A_2 = [\![M_1, M_2^1, \ldots, M_x, \ldots, M_N^1, M_2^2, \ldots, M_K^2 \,|\, \langle a_{l_1}^1, \ldots, a_{l_\alpha}^1, a_{r_1}^2, \ldots, a_{r_\beta}^2 \rangle, \langle b_1, b_2, \ldots, b_m \rangle, \ldots, \langle f_{t_1}^1, \ldots, f_{t_\rho}^1, f_{v_1}^2, \ldots, f_{v_\omega}^2 \rangle, \ldots, \langle w_1, w_2, \ldots, w_n \rangle, \langle d_1, d_2, \ldots, d_q \rangle, \ldots, \langle z_1, z_2, \ldots, z_p \rangle]\!], \quad (12)$$

where $a_{l_i}^1 \in \overline{a^1}, a_{l_i}^1 \notin \overline{a^2}, i \in \langle 1, \ldots, \alpha \rangle; a_{r_j}^2 \notin \overline{a^1}, a_{r_j}^2 \in \overline{a^2}, j \in \langle 1, \ldots, \beta \rangle$; $f_{t_e}^1 \in \overline{f^1}, f_{t_e}^1 \notin \overline{f^2}, e \in \langle 1, \ldots, \rho \rangle$; $f_{v_h}^2 \notin \overline{f^1}, f_{v_h}^2 \in \overline{f^2}, h \in \langle 1, \ldots, \omega \rangle$.

An *exclusive intersection* of two aggregates A_1 and A_2 is the aggregate A_3 which includes only components of A_1 common for both aggregates and is formed according to the following rule:

1. If $A_1 \doteqdot A_2$ then A_3 includes elements of A_1, which are common for both A_1 and A_2, in every tuple:

$$A_1 = [\![M_1, M_2, \ldots, M_N \,|\, \langle a_1^1, a_2^1, \ldots, a_l^1 \rangle, \langle b_1^1, b_2^1, \ldots, b_m^1 \rangle, \ldots, \langle w_1^1, w_2^1, \ldots, w_n^1 \rangle]\!],$$

$$A_2 = [\![M_1, M_2, \ldots, M_N \,|\, \langle a_1^2, a_2^2, \ldots, a_r^2 \rangle, \langle b_1^2, b_2^2, \ldots, b_q^2 \rangle, \ldots, \langle w_1^2, w_2^2, \ldots, w_p^2 \rangle]\!],$$

$$A_3 = A_1 \neg A_2 = [\![M_1, M_2, \ldots, M_N \,|\, \langle a_{l_1}^1, \ldots, a_{l_\alpha}^1 \rangle, \langle b_{m_1}^1, \ldots, b_{m_\gamma}^1 \rangle, \ldots, \langle w_{n_1}^1, \ldots, w_{n_\lambda}^1 \rangle]\!], \quad (13)$$

where $a_{l_i}^1 \in \overline{a^1}, a_{l_i}^1 \in \overline{a^2}, i \in \langle 1, \ldots, \alpha \rangle; b_{m_k}^1 \in \overline{b^1}, b_{m_k}^1 \in \overline{b^2}, k \in \langle 1, \ldots, \gamma \rangle; w_{n_u}^1 \in \overline{w^1}, w_{n_u}^1 \in \overline{w^2}, u \in \langle 1, \ldots, \lambda \rangle$.

2. If $A_1 \mathring{=} A_2$ then A_3 is a null-aggregate:

$$A_1 = [\![M_1^1, M_2^1, \ldots, M_N^1 \,|\, \langle a_1, a_2, \ldots, a_l \rangle, \langle b_1, b_2, \ldots, b_m \rangle, \ldots, \langle w_1, w_2, \ldots, w_n \rangle]\!],$$

$$A_2 = [\![M_1^2, M_2^2, \ldots, M_K^2 \,|\, \langle c_1, c_2, \ldots, c_r \rangle, \langle d_1, d_2, \ldots, d_q \rangle, \ldots, \langle z_1, z_2, \ldots, z_p \rangle]\!],$$

$$A_3 = A_1 \neg A_2 = [\![\varnothing \,|\, \langle \varnothing \rangle]\!] = A_\varnothing. \quad (14)$$

3. If $A_1 \doteq A_2$ then A_3 includes elements of A_1, which are common for both A_1 and A_2, only in tuples of common sets, thus, the number of sets shortens:

$$A_1 = [\![M_1, M_2^1, \ldots, M_x, \ldots, M_N^1 \mid \langle a_1^1, a_2^1, \ldots, a_l^1\rangle,$$
$$\langle b_1, b_2, \ldots, b_m\rangle, \ldots, \langle f_1^1, f_2^1, \ldots, f_t^1\rangle, \ldots, \langle w_1, w_2, \ldots, w_n\rangle]\!],$$

$$A_2 = [\![M_1, M_2^2, \ldots, M_x, \ldots, M_K^2 \mid \langle a_1^2, a_2^2, \ldots, a_r^2\rangle,$$
$$\langle d_1, d_2, \ldots, d_q\rangle, \ldots, \langle f_1^2, f_2^2, \ldots, f_v^2\rangle, \ldots, \langle z_1, z_2, \ldots, z_p\rangle]\!],$$

$$A_3 = A_1 \neg A_2$$
$$= [\![M_1, \ldots, M_x \mid \langle a_{l_1}^1, \ldots, a_{l_\alpha}^1\rangle, \ldots, \langle f_{t_1}^1, \ldots, f_{t_\rho}^1\rangle]\!], \quad (15)$$

where $a_{l_i}^1 \in \overline{a^1}, a_{l_i}^1 \in \overline{a^2}, i \in \langle 1, \ldots, \alpha\rangle; f_{t_e}^1 \in \overline{f^1}$, $f_{t_e}^1 \in \overline{f^2}, e \in \langle 1, \ldots, \rho\rangle$.

Logical operations in the ASA are non-commutative because the sequence order is important in tuples; it differs them from logical operations on sets.

In addition to logical operations, there are *ordering operations* and *arithmetical operations.* Ordering operations include: Sets Ordering, Tuple Ordering, Ascending Sorting, Descending Sorting, Singling, Extraction, Conditional Extraction, Insertion, Conditional Insertion. Arithmetical operations include: Elementwise Addition, Scalar Addition, Elementwise Subtraction, Scalar Subtraction, Elementwise Multiplication, Scalar Multiplication, Elementwise Division, Scalar Division.

The basic *relations* of aggregates are: Is Equal, Is Less, Is Greater, Is Equivalent, Includes, Is Included, Precedes, Succeeds.

The ordering operations, arithmetical operations, and relations of aggregates are not a subject of this paper.

Multi-Image Concept Based on ASA

A multi-image [12] is a complex representation of various data about an object, a subject, or a process of observation which is obtained (measured, generated) in the course of time. Thus, the multi-image in mathematical sense is an aggregate, the first data tuple of which is a tuple of time values. These values can be natural numbers or values of any other type which can be used for evident and monosemantic representation of time.

Let us define the multi-image as it follows:

$$I = [\![T, M_1, \ldots, M_N \mid \langle t_1, \ldots, t_\tau\rangle, \langle a_1, \ldots, a_{n_1}\rangle, \ldots,$$
$$\langle w_1, \ldots, w_{n_N}\rangle]\!],$$

where T is a set of time values.

It is important that $\tau \ge n_i,\ i \in [1, \ldots, N]$. If $\tau = n_i$ then relation between elements of the time tuple and any other tuple is bijective. From practical point of view, it means that all multimodal data have been obtained (measured, generated) simultaneously. Otherwise, some data values are missing (because they have not been measured or generated in certain moments of time) and, thus, they are undefined values.

Logical operations of the ASA can be used for preparing complex data representations (aggregates) which are supposed to be a subject of further data analysis. The precondition is that data sequences are of different modalities and they are recorded with respect to time.

Let us consider an example of the ASA logical operations application for complex representation of multimodal data about a patient's health status. Let these data sequences be obtained from several digital sensors: thermometer, pulsometer, and sphygmomanometer. Then, let the data obtained from these sources belong to the following data sets:

$M_t = [35.0, \ldots, 39.9]$ is a set of temperature values (°C);

$M_p = [50, \ldots, 110]$ is a set of pulse values (bpm);

$M_{sp} = [80, \ldots, 190]$ is a set of systolic pressure values (mmHg);

$M_{dp} = [55, \ldots, 100]$ is a set of diastolic pressure values (mmHg).

We assume that some of measurements are taken simultaneously and the other measurements are taken sequentially (in different days) as well as different combinations of sensors are used. Every case of measuring is presented as a certain aggregate of data. Thus, let us consider the following aggregates of data:

$$A_1 = [\![M_t, M_p \mid \langle 36.4, 36.1, 36.3, 36.2, 36.5, 36.3\rangle,$$
$$\langle 75, 76, 74, 73, 75, 75\rangle]\!],$$

$$A_2 = [\![M_t, M_p \mid \langle 36.5, 36.5, 36.8, 36.6, 36.3, 36.4,$$
$$37.0, 36.5\rangle, \langle 74, 81, 76, 93, 97, 97, 96\rangle]\!],$$

$$A_3 = [\![M_{sp}, M_{dp} \mid \langle 185, 166, 175, 166, 171, 152\rangle, \langle 76, 73,$$
$$74, 73, 71, 76\rangle]\!],$$

$$A_4 = [\![M_t, M_{sp} \,|\, \langle 36.5, 36.5, 36.8, 36.6, 36.3, 36.4, 37.0, 36.5\rangle, \langle 177, 159, 174, 155, 167, 150, 177, 135\rangle]\!].$$

Then, if we need to obtain a complex representation of measurements taken in two consecutive days, we use (1) because the aggregates are compatible:

$$G_1 = A_1 \cup A_2 = [\![M_t, M_p \,|\, \langle 36.4, 36.1, 36.3, 36.2, 36.5, 36.3, 36.5, 36.5, 36.8, 36.6, 36.3, 36.4, 37.0, 36.5\rangle, \langle 75, 76, 74, 73, 75, 75, 74, 81, 76, 93, 97, 97, 96\rangle]\!].$$

The consolidation of data obtained for the same time but from different sensors can be fulfilled according to (2) or (11) because the aggregates are incompatible:

$$G_2 = A_1 \cup A_3 = A_1 \Delta A_3 = [\![M_t, M_p, M_{sp}, M_{dp} \,|\, \langle 36.4, 36.1, 36.3, 36.2, 36.5, 36.3\rangle, \langle 75, 76, 74, 73, 75, 75\rangle, \langle 185, 166, 175, 166, 171, 152\rangle, \langle 76, 73, 74, 73, 71, 76\rangle]\!].$$

To unite data recorded in two consecutive days when some measurements obtained from the same sensor (temperature) and some of them are of different nature (pulse and blood pressure), we need to use (3) because the aggregates are quasi-compatible:

$$G_3 = A_1 \cup A_4 = [\![M_t, M_p, M_{sp} \,|\, \langle 36.4, 36.1, 36.3, 36.2, 36.5, 36.3, 36.5, 36.5, 36.8, 36.6, 36.3, 36.4, 37.0, 36.5\rangle, \langle 75, 76, 74, 73, 75, 75\rangle, \langle 177, 159, 174, 155, 167, 150, 177, 135\rangle]\!].$$

If we need to find out the most frequently occurring measurement values for both days of measuring, we can prepare the aggregate by using (4):

$$G_4 = A_1 \cap A_2 = [\![M_t, M_p \,|\, \langle 36.4, 36.3, 36.5, 36.3, 36.5, 36.5, 36.3, 36.4, 36.5\rangle, \langle 76, 74, 74, 76\rangle]\!].$$

The fact of receiving heterogeneous data (from different sensors) can be discovered by using (5) or (14). At the same time, common values for the data obtained from the same type of a sensor can be got by using (6):

$$G_5 = A_1 \cap A_4 = [\![M_t \,|\, \langle 36.4, 36.3, 36.5, 36.3, 36.5, 36.5, 36.3, 36.4, 36.5\rangle]\!].$$

Discrepancy in the data recorded on a certain day from the data recorded on another day can be discovered by obtaining the following aggregate according to (7):

$$G_6 = A_1 \setminus A_2 = [\![M_t, M_p \,|\, \langle 36.1, 36.2\rangle, \langle 75, 73, 75, 75\rangle]\!].$$

To algorithmizes receiving the date from certain sensors from several available, we can use (8):

$$G_7 = A_1 \setminus A_3 = A_1 = [\![M_t, M_p \,|\, \langle 36.4, 36.1, 36.3, 36.2, 36.5, 36.3\rangle, \langle 75, 76, 74, 73, 75, 75\rangle]\!].$$

The use of (9) allows us to compose the aggregate for discovering discrepancy in specific data (temperature) recorded on a particular day with keeping the record of other data (pulse) which was not measured at another medical monitoring event (when blood pressure was measured instead):

$$G_8 = A_1 \setminus A_4 = [\![M_t, M_p \,|\, \langle 36.1, 36.2\rangle, \langle 75, 76, 74, 73, 75, 75\rangle]\!].$$

If we need to discover discrepancy in two data sequences recorded on different days, then we should use (10):

$$G_9 = A_1 \Delta A_2 = [\![M_t, M_p \,|\, \langle 36.1, 36.2, 36.8, 36.6, 37.0\rangle, \langle 75, 73, 75, 75, 81, 93, 97, 97, 96\rangle]\!].$$

The use of (12) enables composing the aggregate for discovering discrepancy in specific data (temperature) recorded on different days with keeping the record of all other data (pulse and blood pressure) which was measured in one medical monitoring event and which was not measured at another medical monitoring event correspondingly:

$$G_{10} = A_1 \Delta A_4 = [\![M_t, M_p, M_{sp} \,|\, \langle 36.1, 36.2, 36.8, 36.6, 37.0\rangle, \langle 75, 76, 74, 73, 75, 75\rangle, \langle 177, 159, 174, 155, 167, 150, 177, 135\rangle]\!].$$

To find out the values obtained on the first day of measuring, which are the most frequently occurring in measurements on both days, we can prepare the aggregate by using (13):

$$G_{11} = A_1 \neg A_2 = [\![M_t, M_p \,|\, \langle 36.4, 36.3, 36.5, 36.3\rangle, \langle 76, 74\rangle]\!].$$

For a similar task, by using (15), we can obtain the data received on the first of two days from the same type of a sensor and discard data of other types:

$$G_{12} = A_1 \neg A_4 = [\![M_t \,|\, \langle 36.4, 36.3, 36.5, 36.3\rangle]\!].$$

Thus, we can compose different aggregates depending on a task, modality of data, recording time, type of data source, and other conditions.

In the examples given above, the time is present indirectly. However, to compose the multi-image we need to add time values in the aggregates. If $T = [1,\ldots,31]$ is a set of time values (days of a certain month) then the multi-image of aggregate A_1 can be the following:

$$I_1 = [\![T, M_t, M_p \mid \langle 2,3,7,11,14,20\rangle, \langle 36.4, 36.1, 36.3, 36.2, 36.5, 36.3\rangle, \langle 75,76,74,73,75,75\rangle]\!].$$

The practical meaning of this example is that a patient's health status has being monitored e.g. from June 2 to June 20. In total, the patient's temperature and heart beat rate were measured 6 times (on June 2, 3, 7, 11, 14, and 20). Every time both parameters have been measured. For instance, on June 11, the patient had temperature of 36.2 °C and pulse of 73 bpm. In general, the multi-image gives an overall view on the patient's health status and if there is a range of different observations being measured during long time and stored in a medical archive then it can be a proper data set for prediction of future health problems and any other either retrospective or proactive investigations.

We suppose that further data processing can be carried out by using a domain-specific programming language, such as ASAMPL [12].

Conclusions

The algebraic system of aggregates is a tool for complex data representation. It enables multimodal data processing by using a range of operations and relations. In particular, logical operations presented in this paper allow to construct different compositions of multimodal data what in its turn enables complex data representation for compound description of objects and processes in different areas including healthcare.

The key difference in the fulfilment of logical operations in the ASA and logical operations on sets is that an aggregate is a form of complex representation of data sequences where the order of data in each sequence of each type is important and has an influence on the order of data in the result of a certain logical operation. It enables using the ASA for description of real-world objects and processes, characteristics of which can be measured by multiple sensors and recorded as multimodal data of the aggregate.

The ASA is the basis for the multi-image concept which enables overall description of an object, a subject, or a process of observation carried out in the course of time.

The further research should be focused on development of methods and algorithms of multimodal data processing based on the operations of the ASA presented in this paper.

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ЛОГІЧНІ ОПЕРАЦІЇ В АЛГЕБРИЧНІЙ СИСТЕМІ АГРЕГАТІВ ДЛЯ ПОДАННЯ ТА ОБРОБЛЕННЯ МУЛЬТИМОДАЛЬНИХ ДАНИХ

Проблематика. Сфера застосувань програмного забезпечення, яке оперує мультимодальними даними, стає все ширшою. Для розроблення ефективних алгоритмів оброблення мультимодальних даних необхідне їх подання як комплексних структур. Існуючі підходи більшістю орієнтовані на незалежне подання даних різної природи, що знижує ефективність їх застосування для комплексного подання мультимодальних даних.

Мета дослідження. Розроблення математичного забезпечення для комплексного подання та оброблення мультимодальних даних у комп'ютерних системах.

Методика реалізації. Алгебрична система агрегатів розроблена для комплексного подання мультимодальних даних. У ній визначено логічні операції, операції впорядкування та арифметичні операції над агрегатами. Логічні операції можуть використовуватися для формування комплексного подання потрібних даних у формі агрегатів для подальшого їх аналізу. Передумовою такого подання є те, що дані мають належати різним модальностям та бути визначеними в часі, тобто являти собою послідовності.

Результати дослідження. Запропоновано та описано логічні операції над агрегатами. Ці операції дають можливість отримувати різноманітні композиції мультимодальних даних, що робить можливим комплексне подання даних для всебічного опису об'єктів та процесів у різноманітних галузях, в тому числі в медицині. Розроблено та представлено концепцію мультиобразу, яка дає змогу комплексно описувати об'єкт, суб'єкт або процес на основі даних, отриманих під час спостереження, яке виконується з урахуванням часу.

Висновки. Алгебрична система агрегатів є засобом для комплексного подання даних. Вона дає змогу оброблювати мультимодальні дані, використовуючи набір операції та відношень. Основною відмінністю між логічними операціями в алгебричній системі агрегатів та логічними операціями над множинами є те, що агрегат являє собою комплексне подання набору даних, представлених у вигляді послідовностей, де порядок слідування елементів кожної модальності є важливим і таким, що впливає на результат операції.

Ключові слова: алгебрична система агрегатів; мультиобраз; мультимодальні дані; логічні операції.

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ЛОГИЧЕСКИЕ ОПЕРАЦИИ В АЛГЕБРАИЧЕСКОЙ СИСТЕМЕ АГРЕГАТОВ ДЛЯ ПРЕДСТАВЛЕНИЯ И ОБРАБОТКИ МУЛЬТИМОДАЛЬНИХ ДАННЫХ

Проблематика. Сфера использования программного обеспечения, оперирующего мультимодальными данными, все больше расширяется. Для разработки эффективных алгоритмов обработки мультимодальных данных необходимо их представление в виде комплексных структур. Существующие подходы в большинстве ориентированы на независимое представление данных разной природы, что снижает эффективность их использования для комплексного представления мультимодальных данных.

Цель исследования. Разработка математического обеспечения для комплексного представления и обработки мультимодальных данных в компьютерных системах.

Методика реализации. Алгебраическая система агрегатов разработана для комплексного представления мультимодальных данных. В ней определены логические операции, операции упорядочивания и арифметические операции над агрегатами. Логические операции могут использоваться для формирования комплексного представления необходимых данных в форме агрегатов для их дальнейшего анализа. Необходимым условием для такого представления является то, что данные должны принадлежать разным модальностям и быть определенными во времени, то есть являться последовательностями.

Результаты исследования. Разработаны и описаны логические операции над агрегатами. Эти операции позволяют получать разнообразные композиции мультимодальных данных, что делает возможным представление данных для всестороннего описания объектов и процессов в различных областях, в том числе в медицине. Разработана и представлена концепция мультиобраза, которая позволяет комплексно описывать объект, субъект или процесс на основе данных, полученных во время наблюдения, осуществляемого с учетом времени.

Выводы. Алгебраическая система агрегатов является средством для комплексного представления данных. Она позволяет обрабатывать мультимодальные данные, используя набор операций и отношений. Основное отличие между логическими операциями в алгебраической системе агрегатов и логическими операциями над множествами состоит в том, что агрегат представляет собой комплексное представление наборов данных, представленных в виде последовательностей, где порядок следования элементов каждой модальности является важным и влияет на результат операции.

Ключевые слова: алгебраическая система агрегатов; мультиобраз; мультимодальные данные; логические операции.

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